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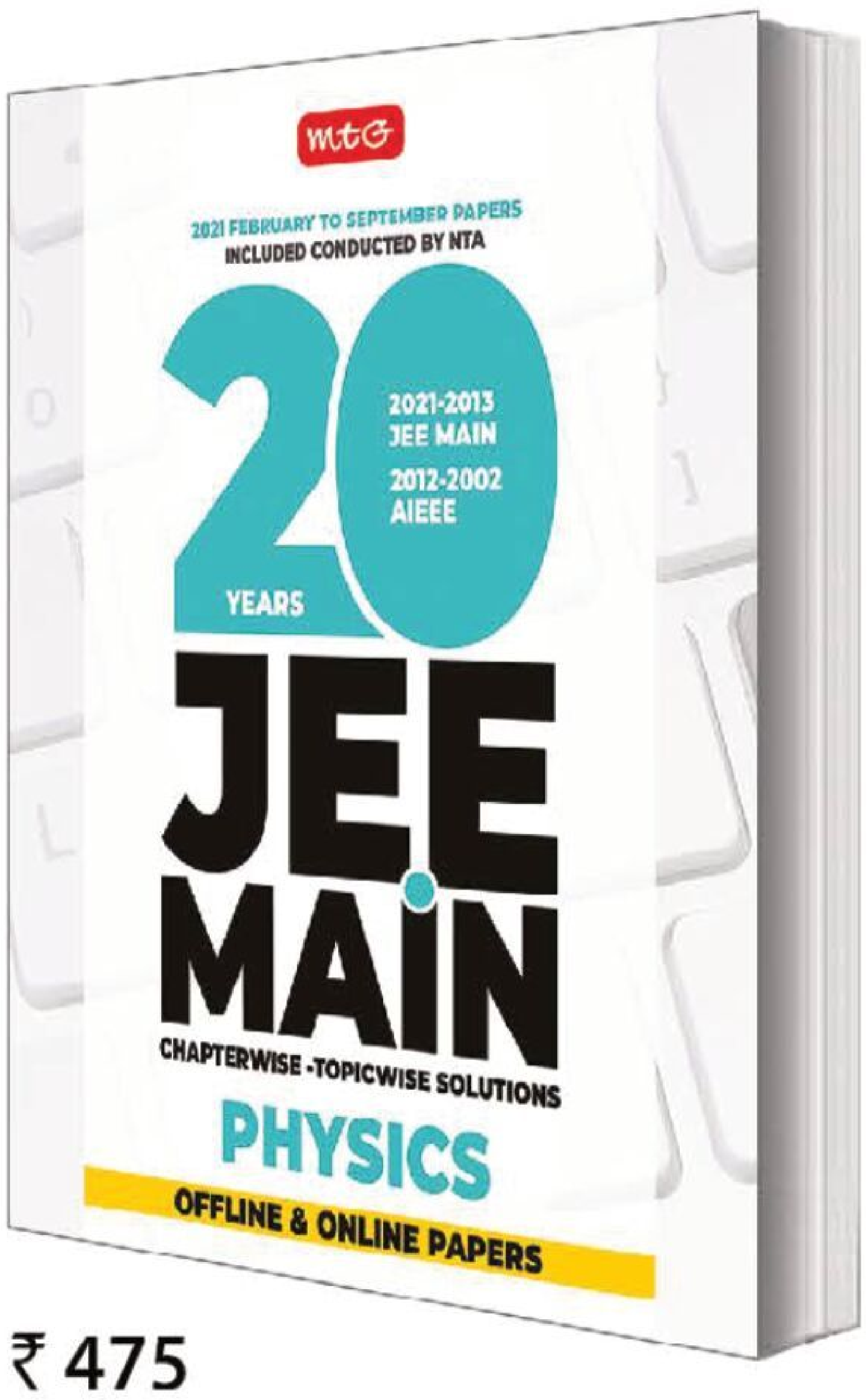
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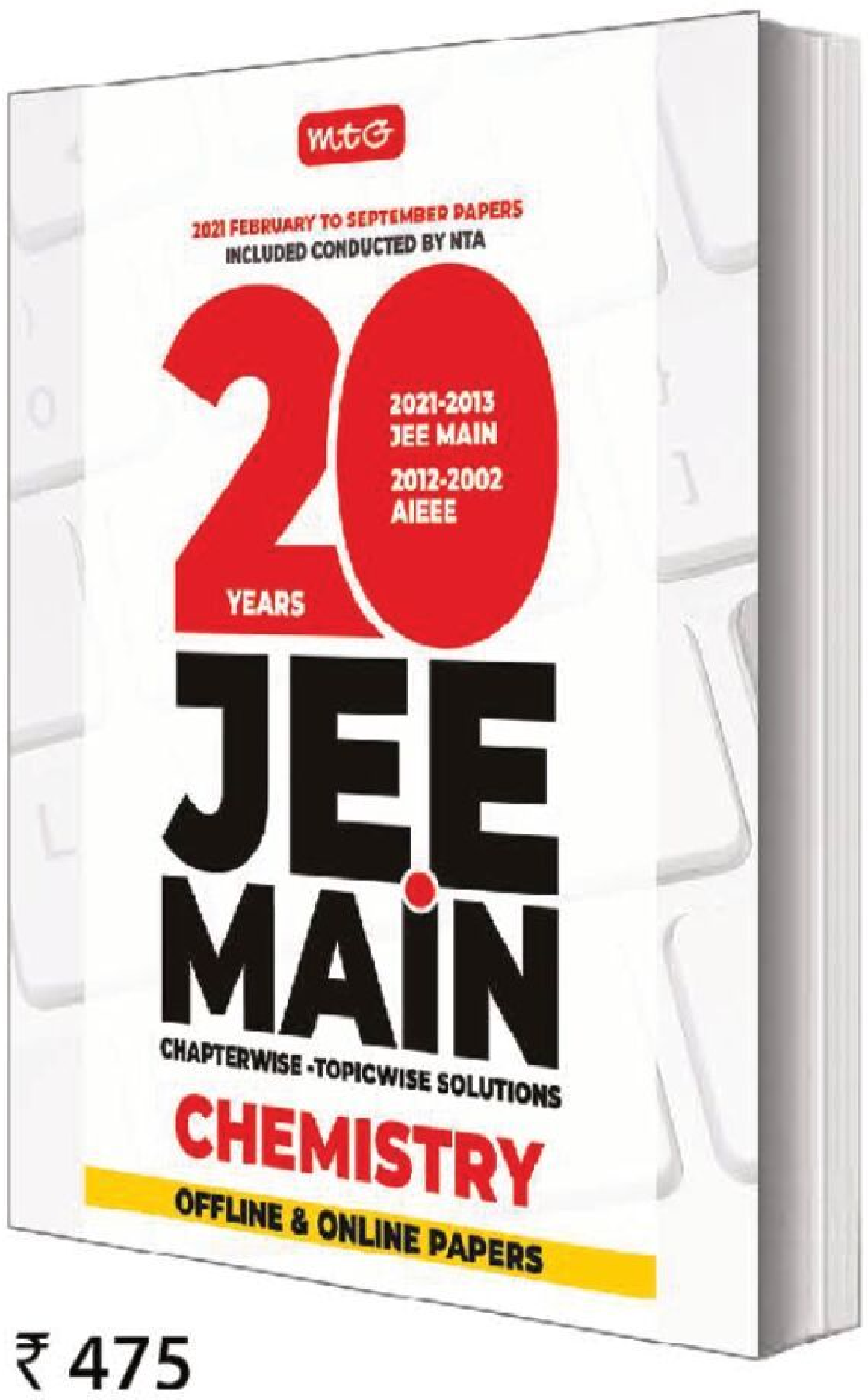
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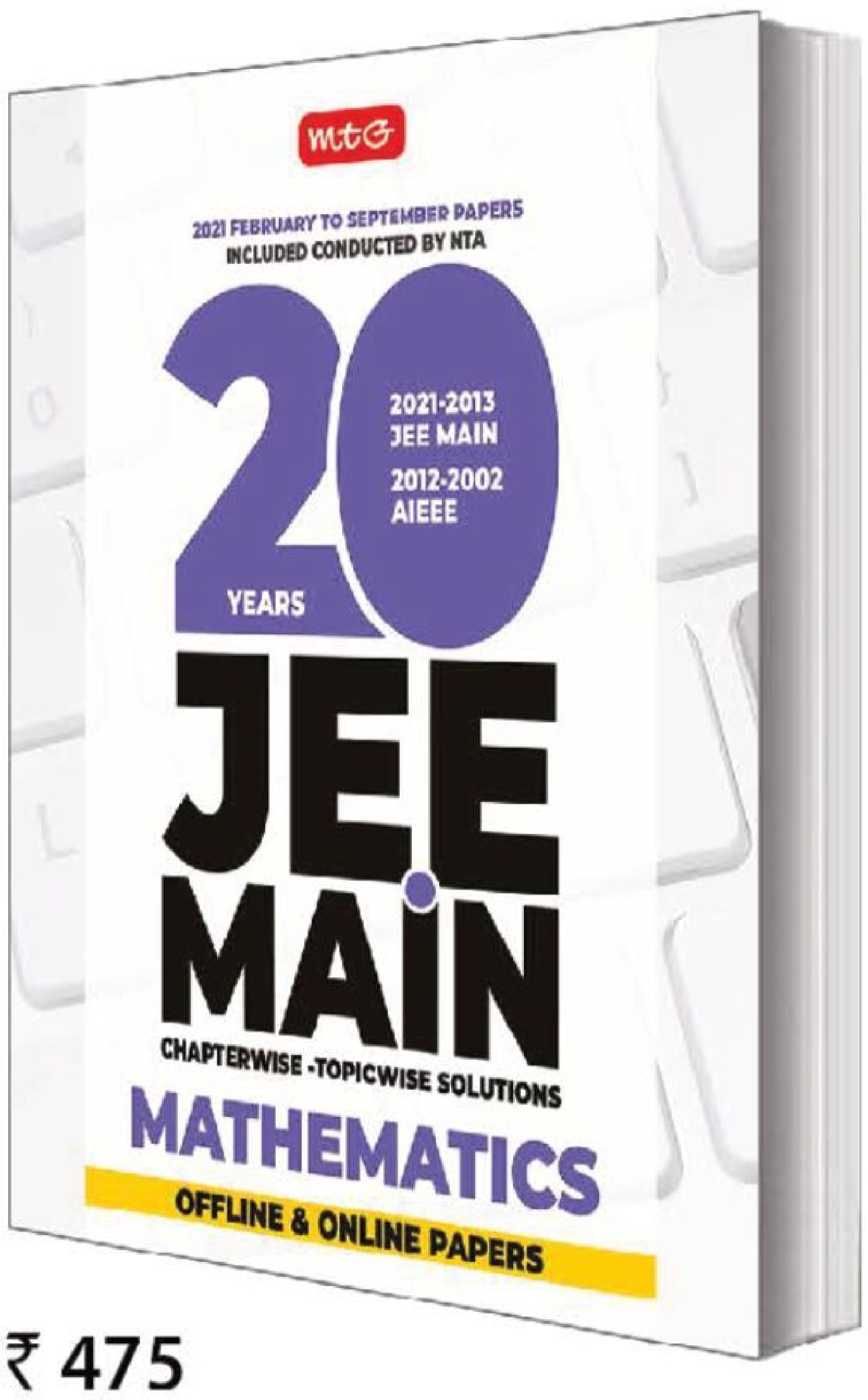
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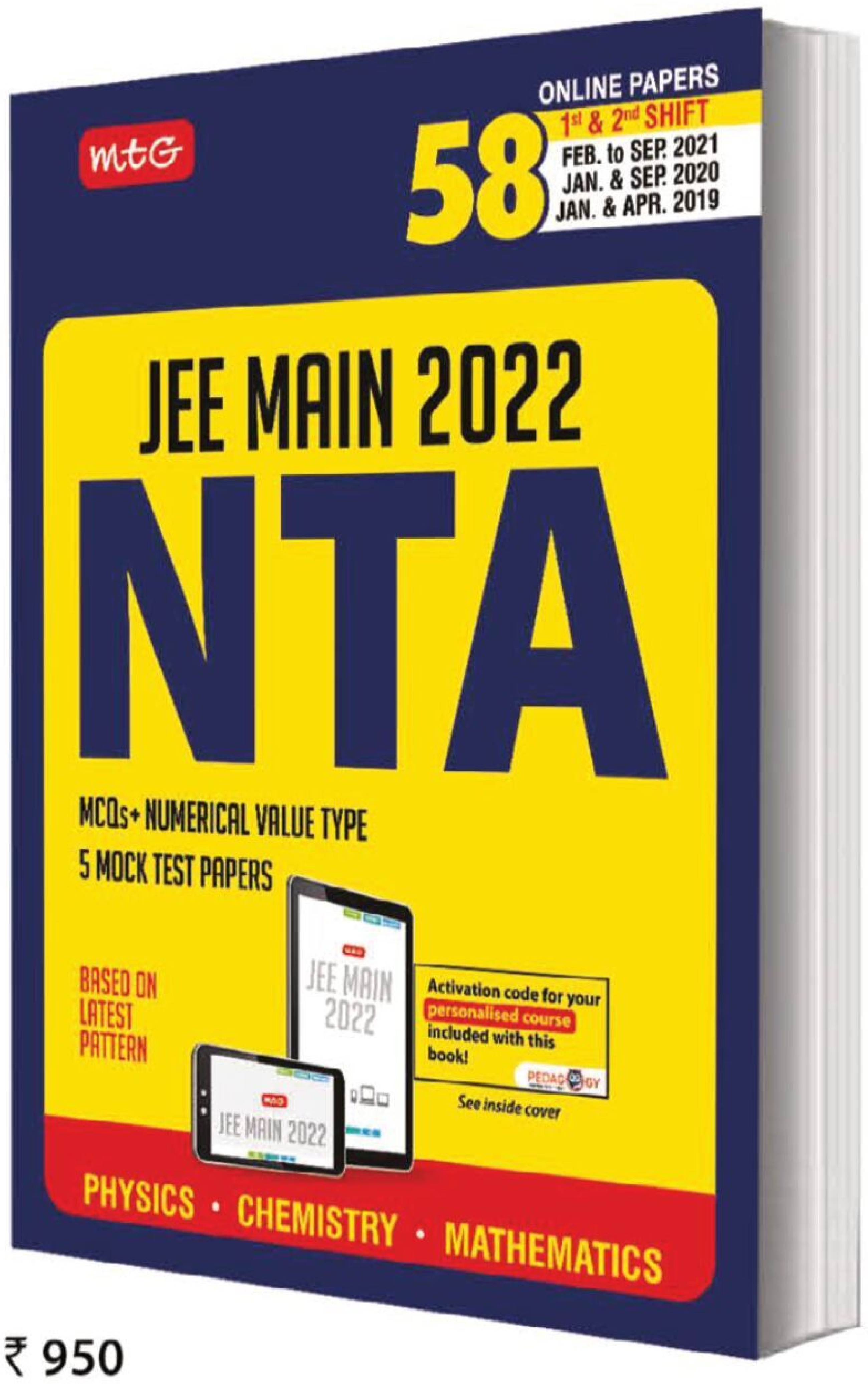
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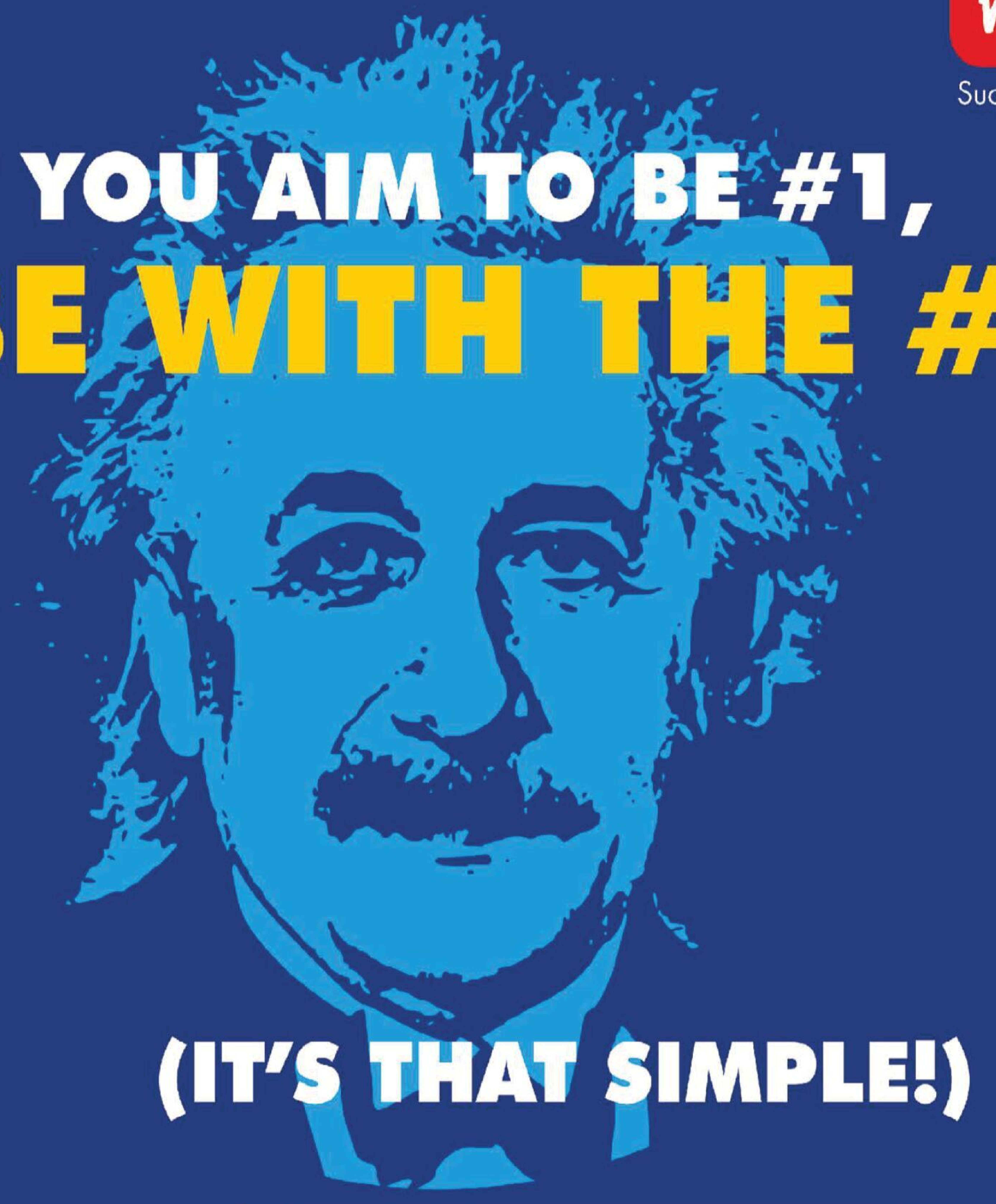

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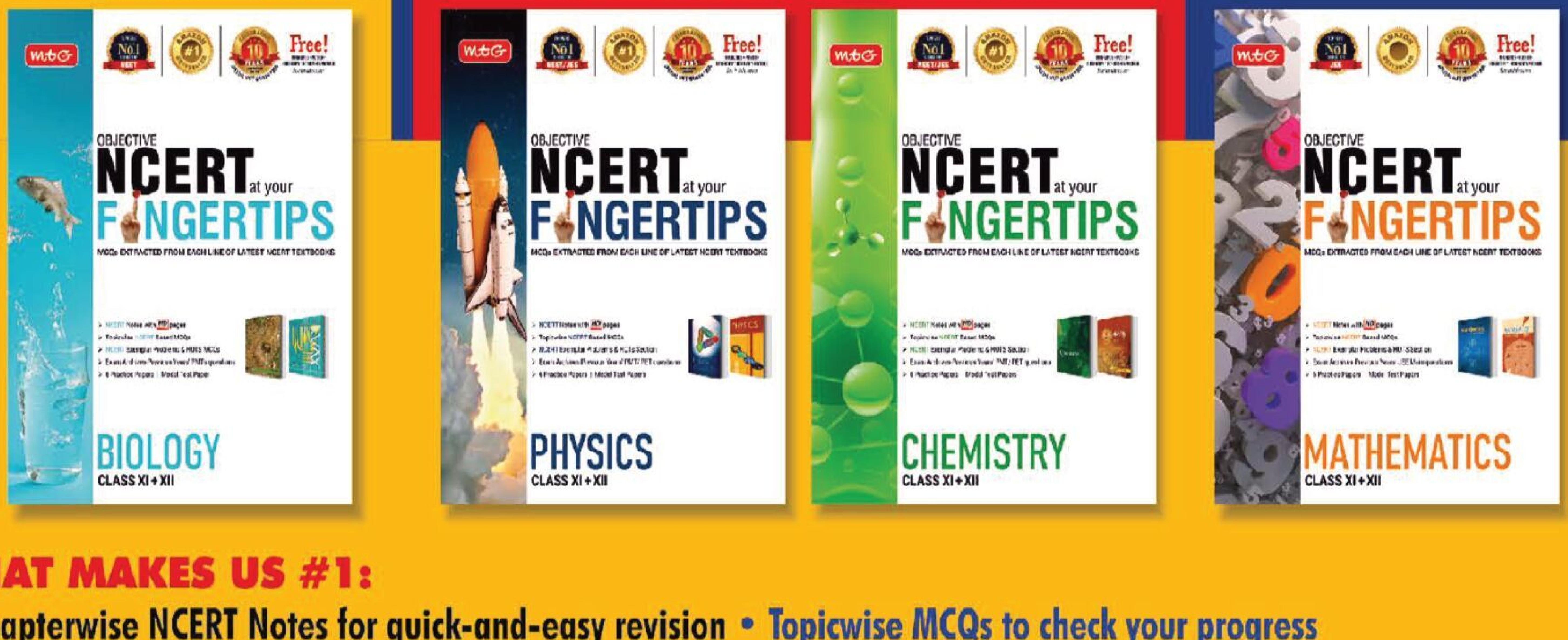
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Tips to make the most of your drop year

Taking a break of a year to study for JEE or NEET is an option steeped in positives. You just need to know what they are. Read on…

Let's address the elephant in the room: Preparing for JEE or NEET exam is a mammoth task. Taking a year off to prepare for it, is equally huge. Take heart though, many students may take a year off to study for two of the most prestigious examinations in the country, which can get them entrance into premier engineering or medical colleges in India. These two entrance exams are typically taken by students while they are in Class 12 or after they clear it. Considering the extensive syllabus, there's a chance that some of you might require a couple of attempts. Whatever be the case, if you do decide to drop a year after your 12th board exam, there are several things you need to take into consideration.

Let's face it

The drop year is going to be frantic. In one year, you will be required to revisit and revise the entire syllabus of Class 11 and Class 12. The good part about this is that you are done with your board exams, and school. Now, you can completely focus on JEE or NEET. But first, pause a bit, and self-evaluate: Assess the problems associated with your exam preparation of previous year/years.

- Was it the lack of consistency, knowledge or time?
- Or was it something else?

You can overcome your initial mistakes with proper planning and time management in the drop year. Here are a few tips that can help you make the best of your drop year.

Analyse your strengths and weaknesses

In the scenario that you have already attempted the exam once before, analyse how you performed in it and what is your current preparation level. Go back to the question paper, and go through it carefully. If you solved a question correctly, was your solution the best one? Was there a more appropriate solution to the question? Could you have solved it faster? If you did not answer a question correctly, find out why. Did you not have enough knowledge of the topic? Did you make a silly mistake? Use this detailed analysis to divide all topics of the syllabus into three segments: Strong, Mediocre and Weak. Doing this is crucial as this will set the tone of your entire preparation effort.

Gather your resources

Next, figure out what resources you need to prepare again:

- **Coaching :** One of the most important decisions for you is whether or not to opt for coaching. That depends on how well you prepared in the previous years and how much guidance you need this year. If you decide to go for expert guidance, there are several options. You can choose the traditional physical coaching or you can opt for a live online course. The advantage of online courses offered by apps such as Amazon Academy is that you learn from expert faculty in the country from the comfort of your home. You don't have to travel daily. That saves time. You also get detailed performance analytics and a wide array of mock tests and practice material alongside the course. While there are several free videos available online, it's better if you have a structured course to help streamline your preparation. Of course, you can use free online videos to know more about a particular topic but to rely on them completely may not be a good move.
- **Books :** A myth-buster for you all — more books do not mean better preparation. Like the saying, 'Too many cooks spoil the broth', having too many books might distract you from focusing on any! If you have prepared for JEE or NEET in Class 11/ 12, you would have a fair idea of the books available for various subjects. Here's a tip: NCERT books are a must, both for JEE and NEET. Apart from that, you could consider investing only in a couple of other books that have relevant material. Ask your teachers to recommend books. Make sure to have a mix of theory and practice books. Don't burden yourself with advanced books unless you feel confident about your preparation and you can go the extra mile.
- **Practice Material :** At the very least, you should consider joining an All India Mock Test (AIMT) series. Most coaching setups, whether online or offline, provide an All India Test series along with their full courses. If not, buy one. Amazon Academy conducts live AIMTs at scheduled intervals designed to replicate the actual exam experience for its subscribers. This not only exposes you

to the nuances of the examination, but also prepares you mentally for D-day. Apart from this, you should have ample material for chapter-wise or topic-wise practice. You can choose to practice with books, coaching modules or independent practice material.

Make a plan

The next step is to formulate an efficient plan. You know the schedule of your lectures by now, if any. Do the following:

- **Make a timetable.** Take the number of hours available in the day and divide your time amongst study, practice, and revision. Set daily goals and ensure that you meet them.
- **Set specific timelines for individual topics and chapters.** You have a year. Plan accordingly. Make sure to allocate some time before the exam for dedicated revision.
- **Be smart about your time allocation.** Now is the time your 'Strong, Mediocre and Weak' areas come into play. When you are setting your timelines, make sure you allot more time to topics you are mediocre or weak in. Mediocre areas may turn into your strengths with relatively lesser effort.
- **Don't forget to schedule time for breaks, sleep, and fun.** While every minute you spend away from books might seem like a minute wasted, it's really not. Your concentration and retention improve considerably when your mind is well-rested. In reality, short breaks may extract more value out of your study hours.

Start from the top

For both JEE and NEET, the most fundamental requirement is that your concepts are clear. If you did not crack the exam in your first attempt, this year you have another chance to go back to the basics.

- **Do not take shortcuts.** Even if you think that you know a topic well, take the time to review it from scratch.
- **Strengthen your basics.** Many advanced topics in JEE and NEET derive from the basic concepts taught in Class 10 or 11. If you don't know them well, the chances of you excelling in the advanced topics are minimal.
- **Ask doubts.** Don't shy away from asking questions. That's what teachers are for. Ask until you have completely understood. Don't leave any room for doubts. Students opting for online apps such as Amazon Academy have various options and opportunities to get their concepts and doubts cleared by expert faculties.

Make Notes

According to the Ebbinghaus Forgetting Curve, you forget about 70 per cent of what you learn in the first 24 hours. This should underline the immense significance of revision in JEE and NEET preparation. One of the most effective ways of absorbing information is by writing it down, this way you are more likely to remember what you studied. Hence, despite having access to all the study materials digitally, including short notes and formula sheets, you should still make your own notes. Colour-code the notes as per your preference. It is not a waste of time as some believe.

Practice

Here's a secret formula to success : Practice!... And you thought there was a shortcut to success! Well, there isn't. Only extensive practice will make you perfect. Your last two to four years of school preparation has already readied you for JEE/NEET exam. All you have to do now is practise old concepts and topics.

- **Employ different techniques for different subjects.** If you are preparing for NEET, Biology requires you to learn a lot of facts and diagrams, and you must improve your retention to perform well. However, a subject such as Physics requires you to work on your conceptual and calculation skills. Derivation is essential for Math. Rely on your teachers to guide you.
- **Take regular mock tests.** You have already covered the syllabus once in your last attempt. Regardless of whether or not you have finished your syllabus, attempt mock tests. This will help you improve your test-taking skills.
- **Take AIMTs.** All India Mock Tests are a must. They help you gauge your preparation as compared to the other candidates. They are timed and you should take them in a simulated exam environment.
- **Solve chapter-wise questions.** Apart from full length tests, you should also solve questions for individual topics as you learn them. This will help you in understanding and retaining the topic better for the exam.

Revise

This one is obvious. You must revise well. Learn from your mistakes in the previous attempt and make sure you don't repeat them in this one. Do remember that revision is not the time to learn new topics you may have missed. Only focus on revising material you have already learnt or practiced as the exam date draws near.

Stay Motivated

Dropping a year is challenging. Along with the strenuous academic demands, you are faced with tremendous mental and emotional pressures too. It is easy to feel exhausted or even burnt out. But, you must work on keeping yourself motivated. Keep your dream college in mind. Ask friends and family to nudge you, to build that focus. The 'light' at the end of the long, arduous study tunnel is just 10-12 months away. Chin up till then!

Disclaimer: The information provided in this article is of a general nature and should not be considered a substitute for professional advice.

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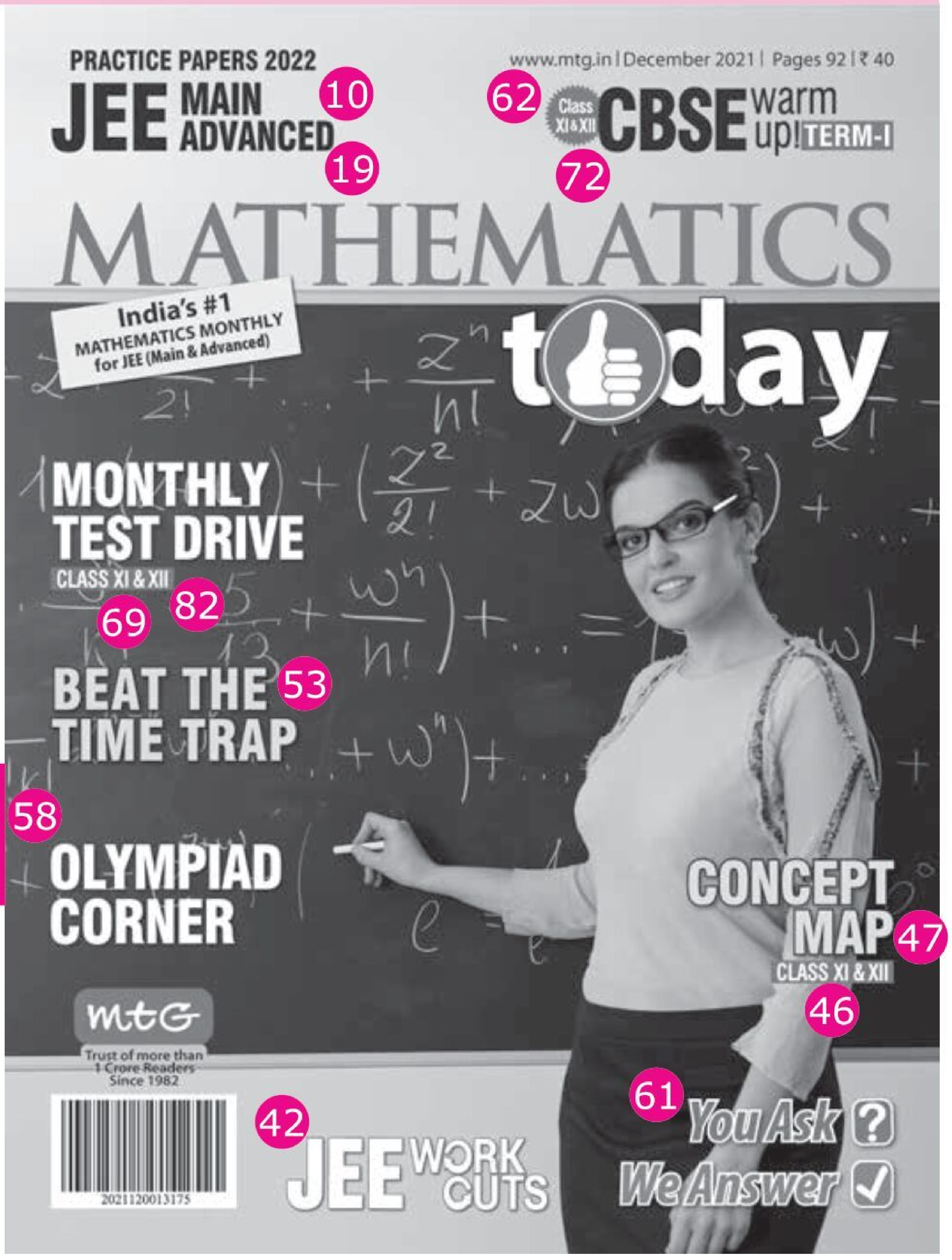
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PRACTICE PAPER 2022

JEE MAIN

Time : 1 hr. **Series - 3**

The entire syllabus of Mathematics of JEE MAIN is being divided into four parts. Each part contains 4 units. There will be a Practice Paper which will be published in the subsequent issues. The syllabus for module break-up is given below:

Units	Topic	Syllabus In Detail
IX	Integral Calculus	Integral as an anti-derivative, Fundamental Integrals involving algebraic, trigonometric, exponential and logarithms functions. Integrations by substitution, by parts and by partial functions. Integration using trigonometric identities. Evaluation of simple integrals of the type $\int \frac{dx}{x^2+a^2}$, $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$, $\int \frac{dx}{\sqrt{a^2-x^2}}$, $\int \frac{dx}{\sqrt{a^2-x^2}}$, $\int \frac{dx}{\sqrt{ax^2+bx+c}}$, $\int \frac{dx}{\sqrt{ax^2+bx+c}}$, $\int \frac{(px+q)\,dx}{ax^2+bx+c}$, $\int \frac{(px+q)\,dx}{\sqrt{ax^2+bx+c}}$ $\int \sqrt{a^2 \pm x^2}\,dx$, $\int \sqrt{x^2-a^2}\,dx$ Integral as limit of a sum. The fundamental theorem of calculus, properties of definite integrals. Evaluation of definite integrals, determining areas of the regions bounded by simple curves in standard from.
X	Differential Equations	Ordinary differential equations, their order and degree, the formation of differential equation, solution of differential equation by the method of separation of variables, solution of a homogeneous and linear differential equation of the type $\frac{dy}{dx} + p(x)y = q(x)$
XI	Co-ordinate Geometry	Cartesian system of rectangular co-ordinates in a plane, distance formula, section formula, locus and its equation, translation of axes, the slope of a line, parallel and perpendicular lines, intercepts of a line on the co-ordinate axis. Straight line : Various forms of equations of a line, intersection of lines, angles between two lines, conditions for concurrence of three lines, the distance of a point from a line, equations of internal and external bisectors of angles between two lines co-ordinate of the centroid, orthocentre and circumcentre of a triangle, equation of the family of lines passing through the point of intersection of two lines. Circle, Conic sections : A standard form of equations of a circle, the general form of the equation of a circle, its radius and central equation of a circle when the endpoints of a diameter are given, points of intersection of a line and a circle with the centre at the origin and condition for a line to be tangent to a circle, equation of the tangent, sections of conics, equation of conic sections (parabola, ellipse and hyperbola) in standard forms, condition for $Y = mx + c$ to be a tangent and point(s) of tangency.
XII	Three Dimensional Geometry	Co-ordinates of a points in space, the distance between two points, section formula, directions ratios and direction cosines, the angle between two intersecting lines. Skew lines, the shortest distance between them and its equation. Equations of a line and a plane in different forms, the intersection of a line and a plane, co-planar lines.

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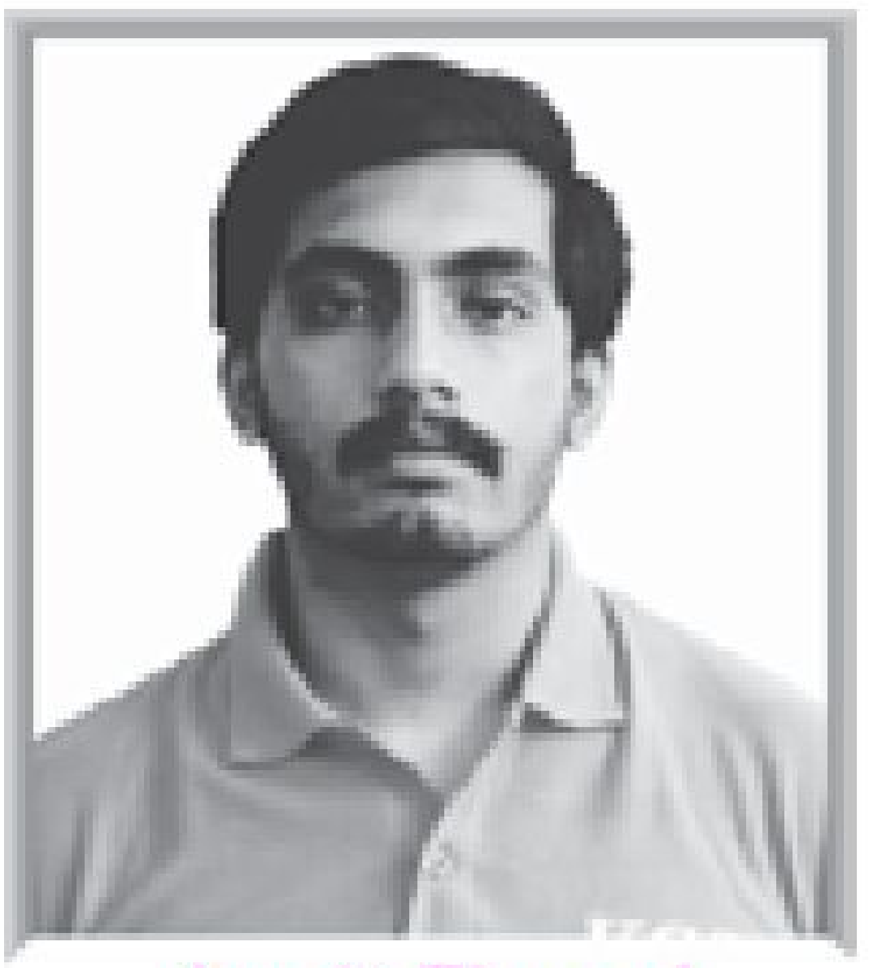

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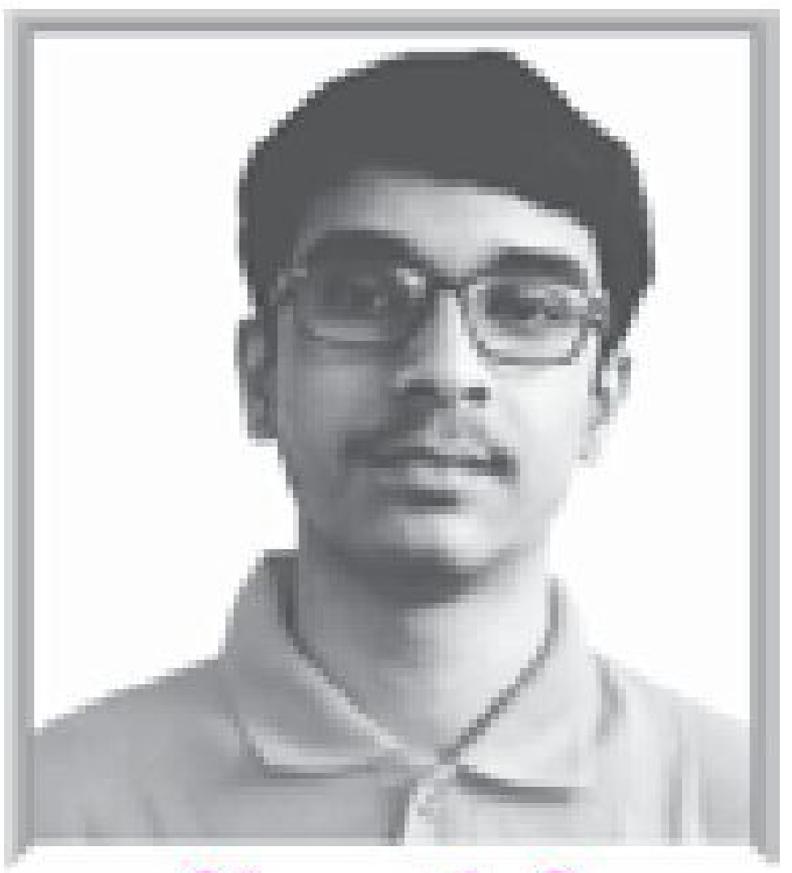

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1. $\int \frac{(x^2-1)}{(x^2+1)\sqrt{x^4+1}}\,dx$, equals

(a) $\sec^{-1}\left(\frac{x^2+1}{x\sqrt{2}}\right)+c$

(b) $\frac{1}{\sqrt{2}}\sec^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)+c$

(c) $\frac{1}{\sqrt{2}}\sec^{-1}\left(\frac{x^2+1}{x\sqrt{2}}\right)+c$

(d) None of these

2. The differential equation of all the circles with centres on the y- axis is

(a) $1 + y_1^2 + yy_2 = 0$ (b) $y_1 + y_1^3 - xy_2 = 0$

(c) $y_1 + y_1^3 + xy_2 = 0$ (d) $1 + y_1^2 - xy_2 = 0$

3. If the slope of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$ be n times the slope of the other, then

(a) $\frac{4h^2}{ab} = \frac{n^2}{(n-1)^2}$ (b) $\frac{4h^2}{ab} = \frac{(n+1)^2}{n}$

(c) $\frac{4h^2}{ab} = \frac{(n-1)^2}{n}$ (d) $\frac{4h^2}{ab} = \frac{n}{(n-1)^2}$

4. A plane passes through the point (0, –1, –2) and is perpendicular to the planes $x + y - 2z = 0$, $2x - y - z = 0$. The distance of the origin from it is

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2

5. If $\int \frac{2e^x + 3e^{-x}}{3e^x + 4e^{-x}}\,dx = Ax + B\log(3e^{2x} + 4) + C$, then

(a) $A = -\frac{3}{4}$, $B = \frac{1}{24}$ (b) $A = \frac{3}{4}$, $B = -\frac{1}{24}$

(c) $A = \frac{1}{4}$, $B = \frac{1}{24}$ (d) $A = -\frac{3}{4}$, $B = \frac{1}{4}$

6. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with

(a) variable radii and fixed centre (0, 1)

(b) variable radii and fixed centre (0, –1)

(c) fixed radius 1 and variable centre on x-axis

(d) fixed radius 1 and variable centre on y-axis

7. The straight line $4x + 3y = p$ is tangent to the circle $x^2 + y^2 + 4\sqrt{3} - 4x - 6y = 0$ then p equals

(a) $10 + 12\sqrt{3}$ (b) $10 - 22\sqrt{3}$

(c) $12 + 10\sqrt{3}$ or $22 - 10\sqrt{3}$

(d) None of these

8. If $2d$ is the shortest distance between the lines $x = 0$, $\frac{y}{b} + \frac{z}{c} = 1$; $y = 0$, $\frac{x}{a} - \frac{z}{c} = 1$, then

(a) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{d}$

(b) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{d^2} = 1$

(c) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} = 1$

(d) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{d^2}$

9. If $\frac{xdy}{dx} + 2y = \ln x$, then $e^2\,y(e) - y(1) =$

(a) $\frac{e^2+1}{2}$ (b) $\frac{e^2+1}{3}$

(c) $\frac{e^2+1}{4}$ (d) $e^2 + 1$

10. The solution of the differential equation $xdy + (x + y)\,dx = 0$ is

(a) $c = \frac{y^2}{2} + xy$ (b) $c = xy + \frac{x^2}{2}$

(c) $c = x + \frac{(xy)^2}{2}$ (d) none of these

11. Equation of pair of tangents to the ellipse $9x^2 + 25y^2 = 225$ from a point (4, 2) is

(a) $9x^2 + 25y^2 - 225 = \left(\frac{36x + 50y - 225}{\sqrt{19}}\right)^2$

(b) $9x^2 + 25y^2 - 225 = \left(\frac{36x + 25y - 225}{15\sqrt{19}}\right)^2$

(c) $\left(\frac{x^2}{25} + \frac{y^2}{9} - 1\right)19 = \left(\frac{36x + 25y - 225}{15}\right)^2$

(d) None of these

12. If a variable plane, at a distance of 3 units from the origin, intersects the coordinate axes at A, B and C, then the locus of the centroid of ΔABC is

(a) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 1$ (b) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 3$

(c) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 9$ (d) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{9}$

13. If $\int_0^{\frac{\pi}{b}} e^{ax}\sin bx\,dx = \frac{\sin\left(\tan^{-1}\left(\frac{b}{a}\right)\right)\left[e^{\frac{a\pi}{b}}+1\right]}{\sqrt{a^2+b^2}}$, then $\int_0^{\pi} e^{x}\sin x\,dx$ equals

(a) $\frac{e^{\pi}+1}{2}$ (b) $\frac{e^{\pi}-1}{2}$

(c) e^{π} (d) $2\,e^{\pi}$

14. Let $P(a\sec\theta, b\tan\theta)$ and $Q(a\sec\phi, b\tan\phi)$, where $\theta+\phi=\frac{\pi}{2}$, be two points on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If (h, k) is the point of intersection of normals at P and Q, then k is

(a) $\frac{a^2+b^2}{a}$ (b) $\frac{-(a^2+b^2)}{a}$

(c) $\frac{a^2+b^2}{b}$ (d) $\frac{-(a^2+b^2)}{b}$

15. The equation of hyperbola referred to its axes as axes of coordinate whose distance between the foci is 20 and eccentricity equals $\sqrt{2}$ is

(a) $x^2-y^2=25$ (b) $x^2-y^2=50$

(c) $x^2-y^2=125$ (d) $x^2+y^2=25$

16. If the lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are perpendicular, then

(a) $ab' + bc' + 1 = 0$ (b) $bb' + cc' + 1 = 0$

(c) $aa' + cc' + 1 = 0$ (d) $aa' + bb' + 1 = 0$

17. If $I_{m-2,n+2}=\int \sin^{m-2}x\cos^{n+2}x\,dx$ and

$$I_{m,n}=\frac{-\sin^{m-1}x\cos^{n+1}x}{n+1}+f(m,n)I_{m-2,n+2},$$

then $f(5, 4)$ is equal to

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

18. The solution curve of $\frac{dy}{dx}=\frac{y^2-2xy-x^2}{y^2+2xy-x^2}$, $y(-1) = 1$ is a/an

(a) straight line (b) parabola

(c) circle (d) ellipse

19. If the normal at an end of a latus rectum passes through the extremity of the minor axis then eccentricity of the ellipse is given by

(a) $e^2 + e + 1 = 0$ (b) $e^4 + e^2 + 1 = 0$

(c) $e^4 - e^2 - 1 = 0$ (d) $e^4 + e^2 - 1 = 0$

20. The length of the perpendicular from $O(0,0,0)$ to the plane passing through a point $\vec{c}$ and containing the line $\vec{r}=\vec{b}+\mu\vec{a}$ is

(a) $\frac{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}{\left|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|}$ (b) $\frac{\left[\vec{c}\,\vec{b}\,\vec{a}\right]}{\left|\vec{b}\times\vec{a}+\vec{a}\times\vec{c}\right|}$

(c) $\frac{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}{\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}\right|}$ (d) $\frac{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}{\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|}$

NUMERICAL ANSWER TYPE

21. If $\int\sqrt{1+\operatorname{cosec}x}\,dx = k\sin^{-1}\sqrt{\sin x}+c$, then $k =$ ________.

22. If the ordinate $x = a$ divides the area bounded by the curve $y = 1+\frac{8}{x^2}$, the x-axis and the ordinate $x = 2$ and $x = 4$ into two equal parts, then $a = \lambda\sqrt{2}$, then λ equals ________.

23. From a point P, tangents are drawn to the ellipse $x^2 + 2y^2 = 2$. If the chord of contact is a normal chord of the ellipse, then the locus of P is $\frac{8}{x^2}+\frac{1}{y^2}=\lambda$, where λ is ________.

24. The angle between the planes $\vec{r}\cdot\left(\hat{i}+2\hat{j}+\hat{k}\right)=4$ and $\vec{r}\cdot\left(-\hat{i}+\hat{j}+2\hat{k}\right)=9$ is $\frac{\pi}{k}$, where $k =$ ________.

25. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos 6x+6\cos 4x+15\cos 2x+10}{\cos 5x+5\cos 3x+10\cos x}dx=$ ________.

26. If the function $y = e^{4x} + 2e^{-x}$ is a solution of the differential equation $\frac{\frac{d^3y}{dx^3}-13\frac{dy}{dx}}{y}=4K$, then the value of K is ________.

27. If the tangents from a point P to the parabola $y^2 = 4x$ form a triangle of unit area with the tangent at the vertex, then the locus of P is $x^2(y^2 - 4x) = \lambda$, where λ is ________.

28. A line from the origin meets the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively. If length $PQ = d$, then d^2 is ________.

29. Area bounded by the curves $\frac{y}{x} = \log x$ and $\frac{y}{2} = -x^2 + x$ equals ________.

30. If $6 + f''(x) + f'(x) = x^2 + f^2(x)$ is the differential equation of a curve and let P be the point of minima of this curve then the number of tangents which can be drawn from P to the circle $x^2 + y^2 = 4$ is ________.

SOLUTIONS

1. (c) : $\int \frac{x^2 - 1}{(x^2+1)\sqrt{x^4+1}}\,dx$

$$= \int \frac{x^2(1 - 1/x^2)}{x^2(x + 1/x)\sqrt{x^2 + 1/x^2}}\,dx$$

$$= \int \frac{(1 - 1/x^2)}{(x + 1/x)\sqrt{(x + 1/x)^2 - 2}}\,dx$$

$$= \int \frac{dt}{t\sqrt{t^2 - 2}} \qquad \text{[Substituting } x + 1/x = t\text{]}$$

$$= \int \frac{dt}{t\sqrt{t^2 - (\sqrt{2})^2}} = \frac{1}{\sqrt{2}}\sec^{-1}\left(\frac{t}{\sqrt{2}}\right) + c$$

$$= \frac{1}{\sqrt{2}}\sec^{-1}\left(\frac{x^2+1}{x\sqrt{2}}\right) + c$$

2. (b): $x^2 + (y - k)^2 = r^2$

Differentiating, $x + (y - k)y_1 = 0$

$1 + {y_1}^2 + (y - k)y_2 = 0$

Eliminating $y - k$, $y_1 + {y_1}^3 - xy_2 = 0$.

3. (b): Let the slope of one line be m, then slope of the other line be nm.

$$\therefore \quad m + mn = \frac{-2h}{b} \Rightarrow \quad m = \frac{-2h}{b(n+1)} \qquad \text{...(i)}$$

$$\text{and } (nm)(m) = \frac{a}{b} \quad \Rightarrow \quad m^2 = \frac{a}{nb} \qquad \text{...(ii)}$$

From (i) & (ii), we have, $\frac{4h^2}{b^2(n+1)^2} = \frac{a}{nb}$

$$\Rightarrow \quad \frac{4h^2}{ab} = \frac{(n+1)^2}{n}$$

4. (c) : The normal to the plane is perpendicular to the lines with d.r.'s 1, 1, –2 and 2, –1, –1.

$$\therefore \quad \text{The plane is } \begin{vmatrix} x - 0 & y + 1 & z + 2 \\ 1 & 1 & -2 \\ 2 & -1 & -1 \end{vmatrix} = 0$$

$\Rightarrow x(-3) - (y + 1)(3) + (z + 2)(-3) = 0$ or $x + y + z + 3 = 0$

The distance of O from it is $\frac{3}{\sqrt{1 + 1 + 1}} = \sqrt{3}$.

5. (b): $\int \frac{2e^x + 3e^{-x}}{3e^x + 4e^{-x}}\,dx = \int \frac{2e^{2x} + 3}{3e^{2x} + 4}\,dx$

$$= \int \frac{2e^{2x}}{3e^{2x} + 4}\,dx + \int \frac{3dx}{3e^{2x} + 4}$$

$$= \frac{2}{6}\int \frac{6e^{2x}}{3e^{2x} + 4}\,dx + 3\int \frac{e^{-2x}}{3 + 4e^{-2x}}\,dx$$

$$= \frac{2}{6}\int \frac{6e^{2x}}{3e^{2x} + 4}\,dx + \left(\frac{3}{-8}\right)\int \frac{-8e^{-2x}}{3 + 4e^{-2x}}\,dx$$

$$= \frac{1}{3}\log\left(3e^{2x} + 4\right) + \left(\frac{3}{-8}\right)\log\left(\frac{3e^{2x} + 4}{e^{2x}}\right) + C$$

$$= \frac{1}{3}\log(3e^{2x} + 4) + \left(\frac{3}{-8}\right)\log(3e^{2x} + 4) + \left(\frac{-3}{8}\right)(-2x) + C$$

$$= \frac{3}{4}x - \frac{1}{24}\log(3e^{2x} + 4) + C = Ax + B\log(3e^{2x} + 4) + C$$

$$\therefore \quad A = \frac{3}{4},\ B = -\frac{1}{24}$$

6. (c) : $\frac{ydy}{\sqrt{1 - y^2}} = dx \Rightarrow -\sqrt{1 - y^2} = x + c$

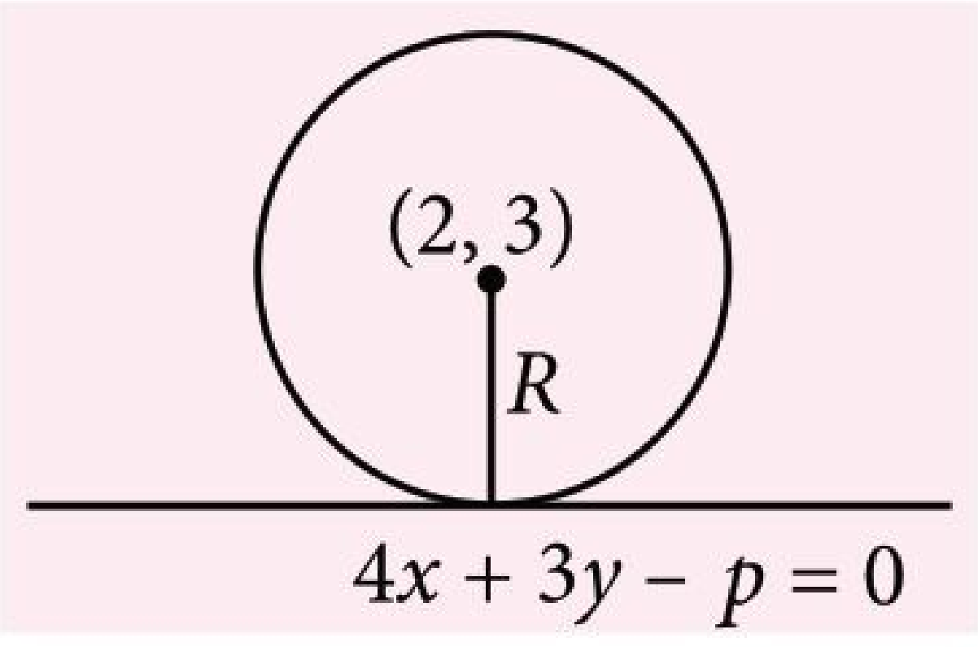
$\Rightarrow$ $1 - y^2 = (x + c)^2 \Rightarrow (x + c)^2 + y^2 = 1$, radius 1 and centre on the x-axis.

7. (c) : Given $x^2 + y^2 - 4x - 6y + 4\sqrt{3} = 0$

$\Rightarrow \quad (x - 2)^2 + (y - 3)^2 = 13 - 4\sqrt{3}$

$\Rightarrow \quad (x - 2)^2 + (y - 3)^2 = \left(2\sqrt{3} - 1\right)^2$

$\Rightarrow$ centre (2, 3) and $R = \left|2\sqrt{3} - 1\right|$



Now $\frac{4 \cdot 2 + 3 \cdot 3 - p}{5} = \pm\left(2\sqrt{3} - 1\right)$

$\Rightarrow \quad 17 - p = \pm(10\sqrt{3} - 5) \ \Rightarrow\ 17 \pm (10\sqrt{3} - 5) = p$

$\Rightarrow \quad p = 17 + 10\sqrt{3} - 5$ or $p = 17 - 10\sqrt{3} + 5$

$\Rightarrow \quad p = 12 + 10\sqrt{3}$ or $p = 22 - 10\sqrt{3}$

8. (d): The lines are $\frac{x}{0} = \frac{y}{b} = \frac{z - c}{-c}$, $\frac{x}{a} = \frac{y}{0} = \frac{z + c}{c}$

$\vec{c} - \vec{a} = -2c\hat{k}$

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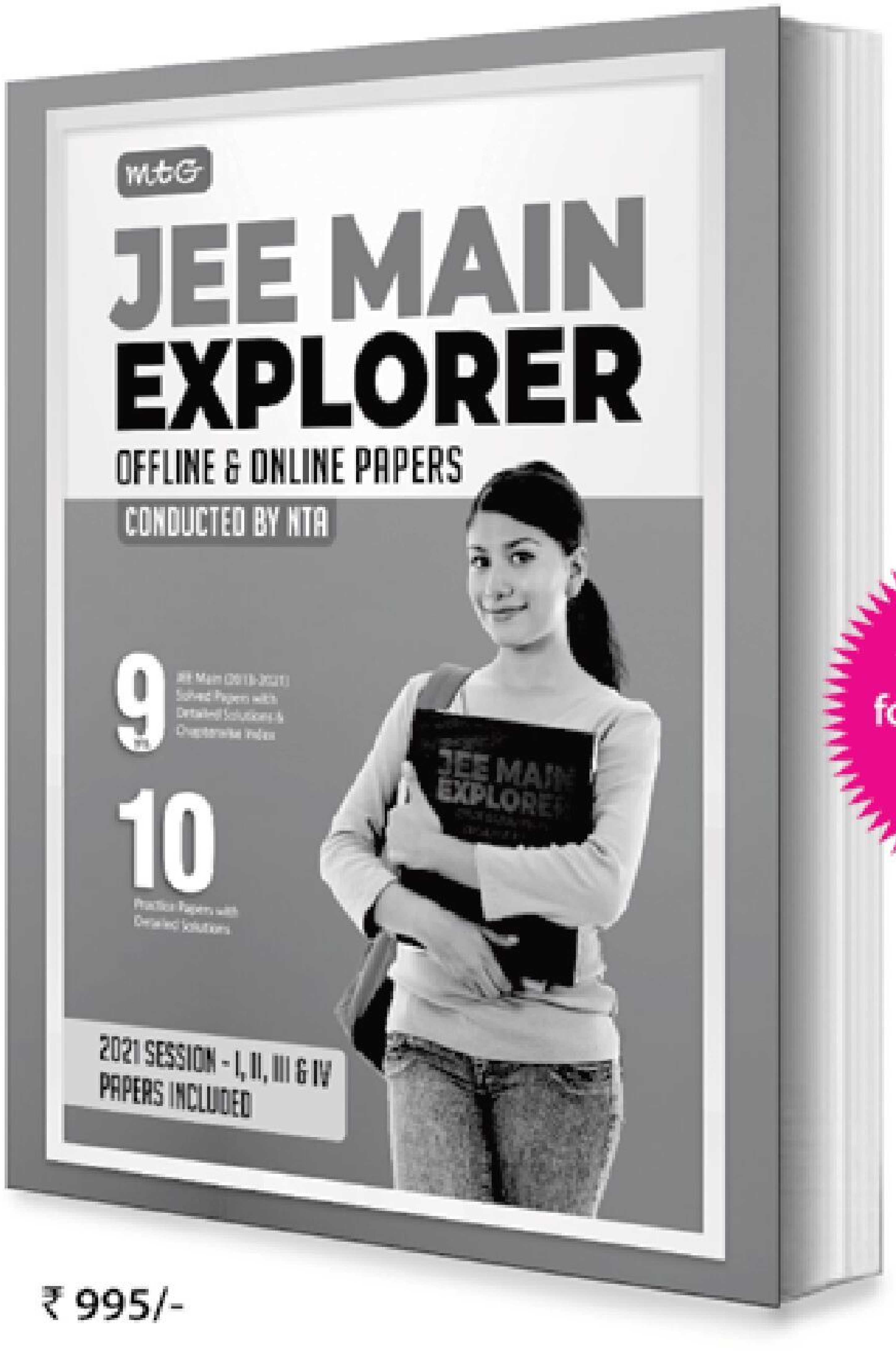
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$$\vec{b}\times\vec{d}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & b & -c\\ a & 0 & c\end{vmatrix}=bc\hat{i}-ca\hat{j}-ab\hat{k}$$

$$\hat{n}=\frac{\vec{b}\times\vec{d}}{|\vec{b}\times\vec{d}|}=\frac{bc\hat{i}-ca\hat{j}-ab\hat{k}}{\sqrt{b^2c^2+c^2a^2+a^2b^2}}$$

Shortest distance $= |(\vec{c}-\vec{a})\cdot\hat{n}|$

$$=\frac{2abc}{\sqrt{b^2c^2+c^2a^2+a^2b^2}}=2d$$

$$\therefore\quad \frac{1}{d^2}=\frac{b^2c^2+c^2a^2+a^2b^2}{a^2b^2c^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}.$$

9. (c) : We have, $\frac{dy}{dx}+\frac{2}{x}y=\frac{\ln x}{x}$.

It is a linear differential equation with I.F. $= e^{\int\frac{2}{x}dx}=x^2$

$\therefore$ Solution is, $yx^2=\int x^2\cdot\frac{\ln x}{x}dx=\int x\ln x\,dx$

$\Rightarrow\ yx^2=\frac{x^2}{2}\ln x-\frac{x^2}{4}+c$

$\therefore\ x=e\Rightarrow e^2y(e)=\frac{e^2}{2}-\frac{e^2}{4}+c=\frac{e^2}{4}+c$

and $x=1\Rightarrow y(1)=-\frac{1}{4}+c$

So, $e^2y(e)-y(1)=\frac{e^2+1}{4}$.

10. (b): Given, $xdy+(x+y)\,dx=0$

$\Rightarrow\ \frac{dy}{dx}=-\left(\frac{x+y}{x}\right)$

Put $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore\ v+x\cdot\frac{dv}{dx}=-(1+v)\Rightarrow x\frac{dv}{dx}=-(1+2v)$

$\Rightarrow\ \frac{dv}{1+2v}=-\frac{dx}{x}\Rightarrow\frac{\log(1+2v)}{2}=\log\left(\frac{k}{x}\right)$

$\Rightarrow\ \log\left(1+\frac{2y}{x}\right)=\log\left(\frac{k^2}{x^2}\right)\Rightarrow\left(1+\frac{2y}{x}\right)=\frac{k^2}{x^2}$

$\Rightarrow\ x^2+2xy=k^2\Rightarrow x^2/2+xy=c$

11. (a): Equation of pair of tangents to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ from (x_1, y_1) is given by $SS'=T^2$

$$\Rightarrow\ \left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)\left(\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1\right)=\left(\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1\right)^2$$

$\therefore$ Using $(x_1, y_1)=(4, 2)$, we have

$$T=\left(\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1\right)=\left(\frac{4x}{25}+\frac{2y}{9}-1\right),$$

$$S=\frac{x^2}{a^2}+\frac{y^2}{b^2}-1=\frac{x^2}{25}+\frac{y^2}{9}-1$$

and $S'=\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1=\left(\frac{16}{25}+\frac{4}{9}-1\right)$

$\therefore$ Required equation of pair of tangents is

$$\left(\frac{x^2}{25}+\frac{y^2}{9}-1\right)\left(\frac{16}{25}+\frac{4}{9}-1\right)=\left(\frac{4x}{25}+\frac{2y}{9}-1\right)^2$$

or, $(9x^2+25y^2-225)=\left(\frac{36x+50y-225}{\sqrt{19}}\right)^2$

12. (a) : Let centroid be (h, k, l).

$\therefore$ x-intercept $=3h$, y-intercept $=3k$, z-intercept $=3l$

Equation of plane is $\frac{x}{3h}+\frac{y}{3k}+\frac{z}{3l}=1$

Distance of plane from (0, 0, 0) is

$$\left|\frac{-1}{\sqrt{\frac{1}{9h^2}+\frac{1}{9k^2}+\frac{1}{9l^2}}}\right|=3\Rightarrow 1=3\left(\frac{1}{3}\right)\sqrt{\frac{1}{h^2}+\frac{1}{k^2}+\frac{1}{l^2}}$$

Thus, locus is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1$

13. (a) : $\int_0^{\frac{\pi}{b}}e^{ax}\sin bx\,dx=\frac{\sin\left(\tan^{-1}\left(\frac{b}{a}\right)\right)\left[e^{\frac{a\pi}{b}}+1\right]}{\sqrt{a^2+b^2}}$

Putting $a=b=1$, we get

$$\int_0^{\pi}e^x\sin x\,dx=\frac{\sin(\tan^{-1}1)(e^{\pi}+1)}{\sqrt{2}}$$

$$=\left(\sin\frac{\pi}{4}\right)\frac{(e^{\pi}+1)}{\sqrt{2}}=\frac{e^{\pi}+1}{2}$$

14. (d): Normal at θ is $\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$ ...(i)

Normal at $\phi=\frac{\pi}{2}-\theta$ is $\frac{ax}{\operatorname{cosec}\theta}+\frac{by}{\cot\theta}=a^2+b^2$ ....(ii)

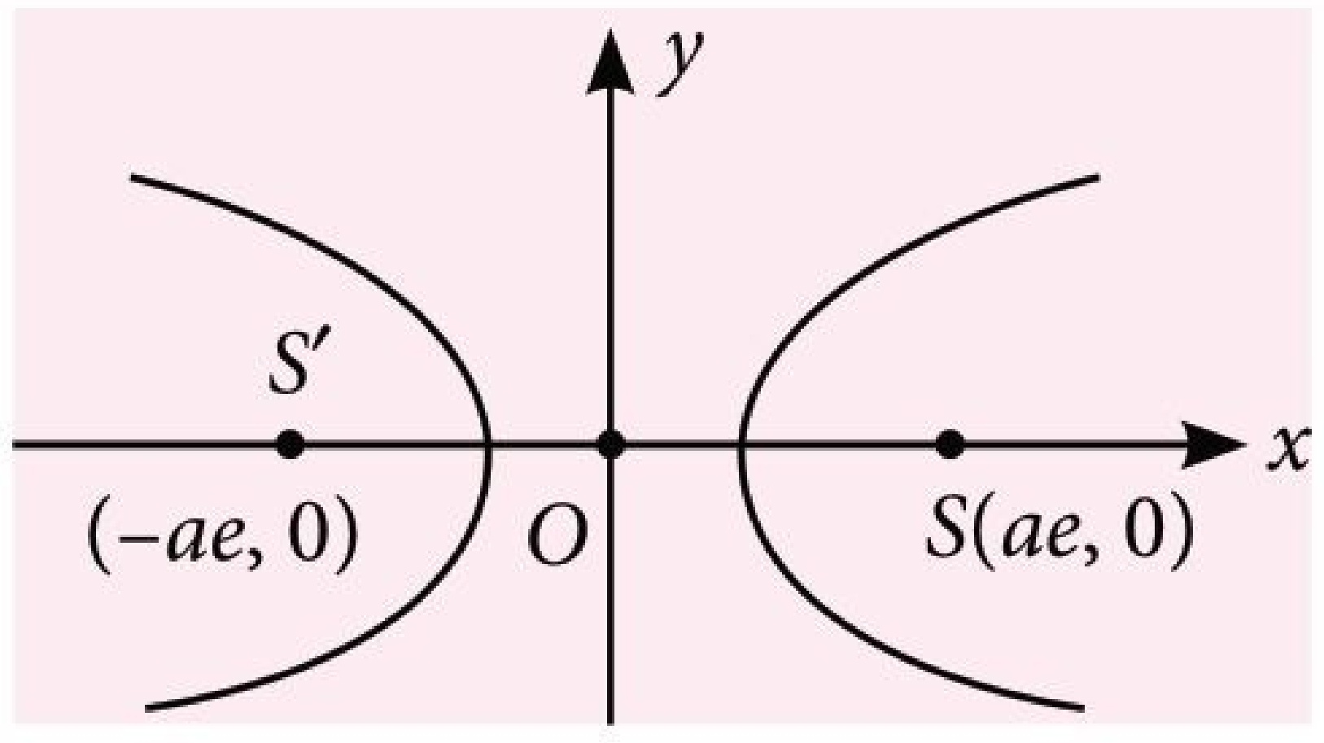
Eliminating x from (i) and (ii), we get

$$by\left(\frac{1}{\sin\theta}-\frac{1}{\cos\theta}\right)=(a^2+b^2)\left(\frac{1}{\cos\theta}-\frac{1}{\sin\theta}\right)$$

$\therefore\ y=\frac{-(a^2+b^2)}{b}$.

15. (b): Given $2ae=20$ and $e=\sqrt{2}$

$\therefore\ a=\frac{20}{2e}=5\sqrt{2}$



Again $b^2 = a^2(e^2 - 1)$

$\therefore\quad b^2 = 25 \times 2(2 - 1) = 50$

$\therefore$ Equation of hyperbola is $x^2 - y^2 = 50$

16. (c) : Symmetrical forms of given equations are

$$\frac{x-b}{a}=\frac{y-0}{1}=\frac{z-d}{c}$$

and $\dfrac{x-b'}{a'}=\dfrac{y-0}{1}=\dfrac{z-d'}{c'}$

Now, condition of perpendicularity is,

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow\quad aa' + cc' + 1 = 0$

17. (d): Let $A = \sin^{m-1} x \cos^{n+1} x$

$$\therefore\quad \frac{dA}{dx}=\cos^{n+1}x(m-1)\sin^{m-2}x\cos x + \sin^{m-1}x(n+1)\cos^n x(-\sin x)$$

$$\Rightarrow\quad \frac{dA}{dx}=(m-1)\sin^{m-2}x\cos^{n+2}x-(n+1)\sin^m x\cos^n x$$

$$\Rightarrow (n+1)\sin^m x\cos^n x=-\frac{dA}{dx}+(m-1)\sin^{m-2}x\cos^{n+2}x$$

$$\Rightarrow\quad (n+1)I_{m,n}=-A+(m-1)I_{m-2,n+2}\quad \text{(on integrating)}$$

$$\Rightarrow\quad (n+1)I_{m,n}=-\sin^{m-1}x\cos^{n+1}x+(m-1)I_{m-2,n+2}$$

$$\Rightarrow\quad I_{m,n}=\frac{m-1}{n+1}I_{m-2,n+2}-\frac{\sin^{m-1}x\cos^{n+1}x}{n+1}$$

$$I_{m,n}=f(m,n)I_{m-2,n+2}-\frac{\sin^{m-1}x\cos^{n+1}x}{n+1}$$

$$\therefore\quad f(m,n)=\frac{m-1}{n+1}\quad \therefore\quad f(5,4)=\frac{5-1}{4+1}=\frac{4}{5}$$

18. (a): Put $y = vx$, then given equation becomes

$$\frac{xdv}{dx}+v=\frac{v^2-2v-1}{v^2+2v-1}$$

$$\Rightarrow\quad \frac{xdv}{dx}=\frac{v^2-2v-1}{v^2+2v-1}-v=-\frac{(v^2+1)(v+1)}{v^2+2v-1}$$

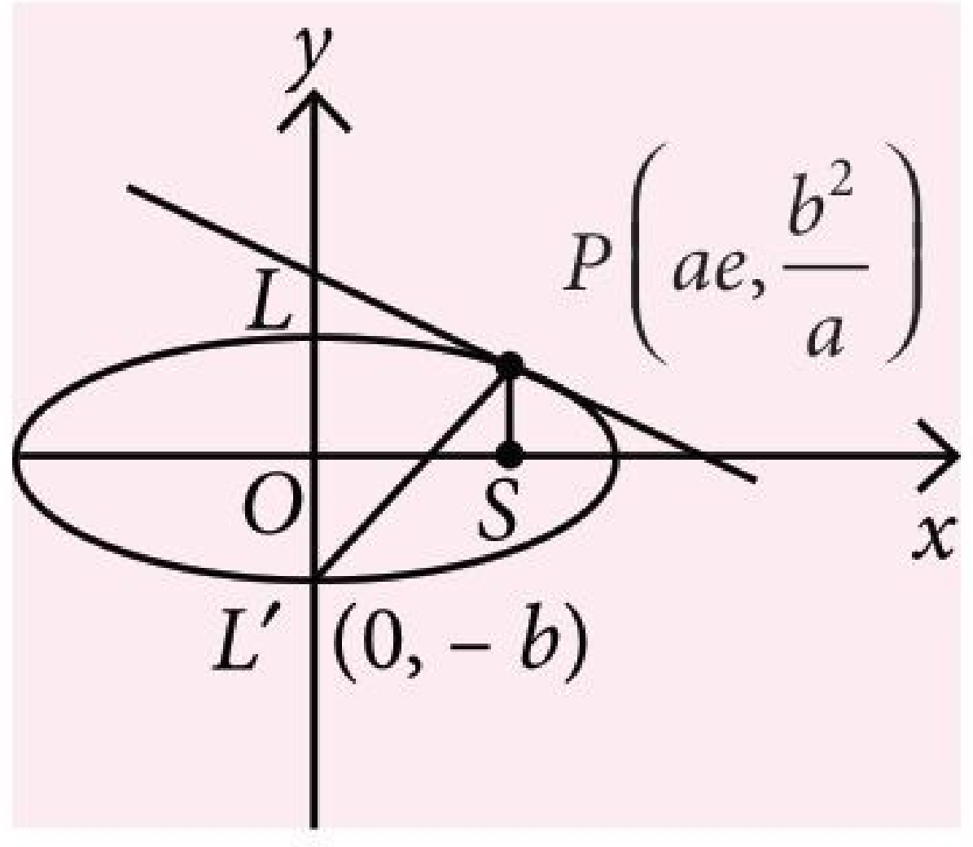
$$\Rightarrow\quad \frac{dx}{x}=\frac{-(v^2+2v-1)dv}{(v^2+1)(v+1)}=\left(\frac{1}{v+1}-\frac{2v}{v^2+1}\right)dv$$

$$\Rightarrow\quad \ln x=\ln\left(\frac{v+1}{v^2+1}\right)+\ln c_1 \Rightarrow x=\frac{c_1(x+y)x}{(x^2+y^2)}$$

$\Rightarrow\quad c(x^2 + y^2) = x + y$

When $x = -1, y = 1 \Rightarrow c = 0 \Rightarrow x + y = 0$.

19. (d): Let equation of ellipse be $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ and $P\left(ae,\dfrac{b^2}{a}\right)$ be one end of a latus rectum through $S(ae, 0)$.



Now equation of normal at P is

$$\frac{a^2x}{ae}-\frac{b^2y}{b^2/a}=a^2-b^2$$

$$\Rightarrow\quad \frac{ax}{e}-ay=a^2e^2$$

Now this normal passing through $L'(0, -b)$.

$\therefore\quad b = ae^2 \Rightarrow b^2 = a^2e^4$

$\therefore\quad a^2(1 - e^2) = a^2e^4 \Rightarrow e^4 + e^2 - 1 = 0$

20. (b): Given plane passes through $\vec{c}$ and is parallel to the vector $\vec{b}-\vec{c}$ and $\vec{a}$ so it is normal to $\left(\vec{b}-\vec{c}\right)\times\vec{a}$.

Hence its equation is $\left(\vec{r}-\vec{c}\right)\cdot\left[\left(\vec{b}-\vec{c}\right)\times\vec{a}\right]=0$

i.e. $\vec{r}\cdot\left(\vec{b}\times\vec{a}+\vec{a}\times\vec{c}\right)=\left[\vec{c}\,\vec{b}\,\vec{a}\right]$ ...(i)

$\therefore$ Length of ⊥ from $O(0, 0, 0)$ to (i) is

$$\frac{\left|\vec{O}\cdot\left(\vec{b}\times\vec{a}+\vec{a}\times\vec{c}\right)-\left[\vec{c}\ \vec{b}\ \vec{a}\right]\right|}{\left|\vec{b}\times\vec{a}+\vec{a}\times\vec{c}\right|}=\frac{\left[\vec{c}\,\vec{b}\,\vec{a}\right]}{\left|\vec{b}\times\vec{a}+\vec{a}\times\vec{c}\right|}$$

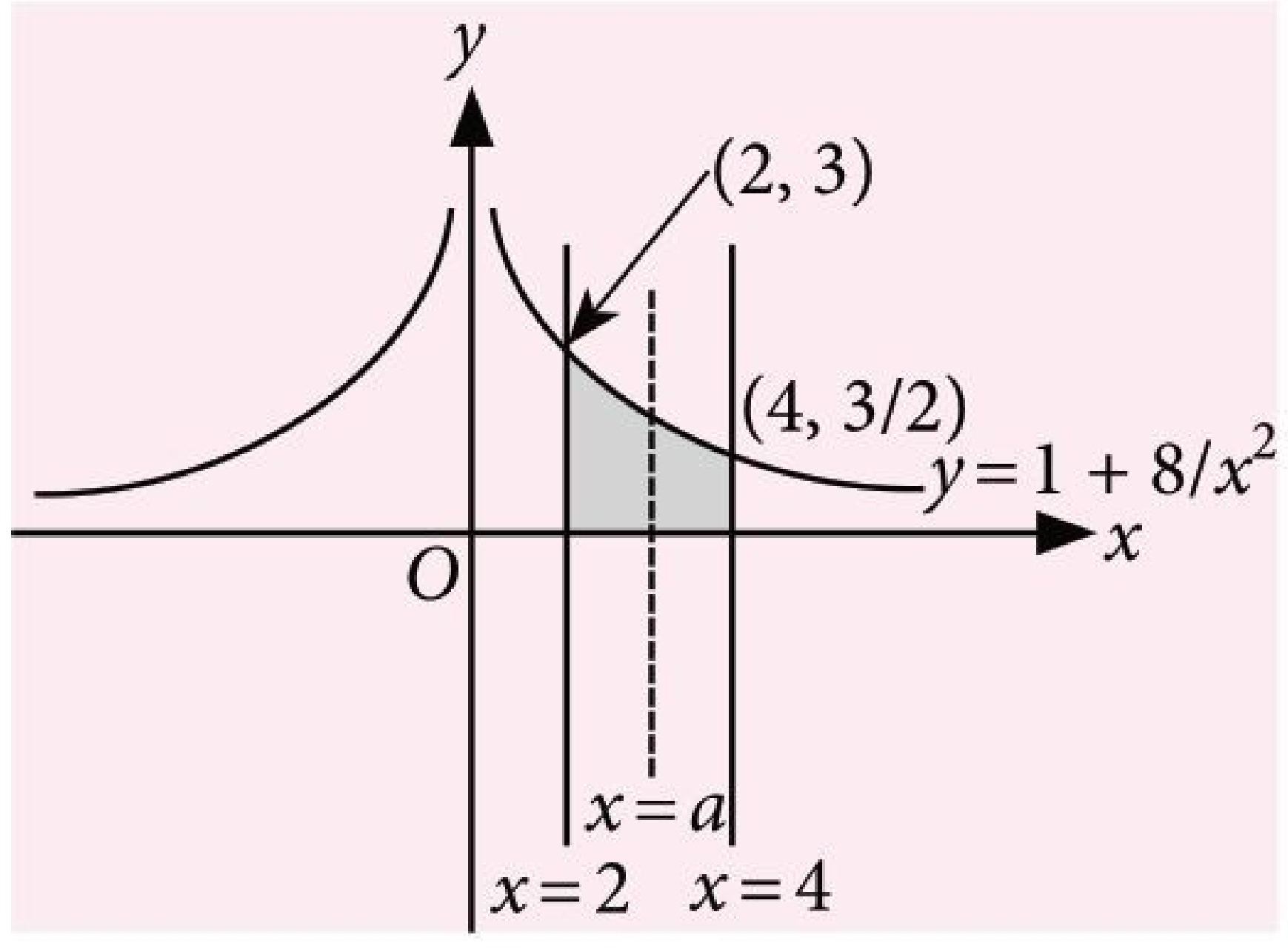
21. (2): $\displaystyle\int\sqrt{1+\operatorname{cosec}x}\,dx=\int\frac{\sqrt{\sin x+1}}{\sqrt{\sin x}}dx$

$$=2\int\frac{\sqrt{t^2+1}}{\sqrt{1-t^4}}dt\quad [\text{Putting } \sin x = t^2 \Rightarrow \cos x\,dx = 2t\,dt]$$

$$=2\int\frac{dt}{\sqrt{1-t^2}}dt=2\sin^{-1}t+c=2\sin^{-1}(\sqrt{\sin x})+c$$

$\therefore\quad k = 2$

22. (2): We have, $\displaystyle\int_2^a\left(1+\frac{8}{x^2}\right)dx=\int_a^4\left(1+\frac{8}{x^2}\right)dx$



$$\Rightarrow\left[a-2-\frac{8}{x}\right]_2^a=\left[4-a-\frac{8}{x}\right]_a^4$$

$$\Rightarrow\quad 2a-6=8\left(\frac{1}{a}-\frac{1}{2}\right)-8\left(\frac{1}{4}-\frac{1}{a}\right)=-6+\frac{16}{a}$$

$\Rightarrow a^2 = 8 \Rightarrow a = 2\sqrt{2}\ \ \therefore\ \lambda = 2$

23. (1): The chord of contact of tangents from $P(x_1, y_1)$ is $xx_1 + 2yy_1 = 2$ ...(i)

The normal chord at $(\sqrt{2}\cos\theta, \sin\theta)$ is

$$\frac{\sqrt{2}x}{\cos\theta} - \frac{y}{\sin\theta} = 1 \qquad ...(ii)$$

Now, (i), (ii) are identical.

$$\therefore \quad \frac{x_1\cos\theta}{\sqrt{2}} = \frac{2y_1\sin\theta}{-1} = 2$$

$$\Rightarrow \quad \cos\theta = \frac{2\sqrt{2}}{x_1},\ \sin\theta = -\frac{1}{y_1}$$

Thus, locus of P is $\frac{8}{x^2} + \frac{1}{y^2} = 1$.

$\therefore \quad \lambda = 1$

24. (3): Angle between two planes will be the angle between their normals.

If $\vec{r}\cdot\vec{n}_1 = d_1$ and $\vec{r}\cdot\vec{n}_2 = d_2$ then

$$\cos\theta = \frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \frac{(\hat{i}+2\hat{j}+\hat{k})\cdot(-\hat{i}+\hat{j}+2\hat{k})}{|\hat{i}+2\hat{j}+\hat{k}||-\hat{i}+\hat{j}+2\hat{k}|}$$

$$\Rightarrow \quad \cos\theta = \frac{-1+2+2}{\sqrt{36}} = \frac{1}{2} \quad \therefore\ \theta = \pi/3 \Rightarrow k = 3$$

25. (4): $\cos 6x + 6\cos 4x + 15\cos 2x + 10$

$= \cos 6x + \cos 4x + 5(\cos 4x + \cos 2x) + 10(\cos 2x + 1)$

$= 2\cos x(\cos 5x + 5\cos 3x + 10\cos x)$

$$\therefore I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\cos x\,dx = 4\int_{0}^{\frac{\pi}{2}} \cos x\,dx = 4.$$

26. (3): $y = e^{4x} + 2e^{-x}$; $y_1 = 4e^{4x} - 2e^{-x}$;

$y_2 = 16e^{4x} + 2e^{-x}$; $y_3 = 64e^{4x} - 2e^{-x}$

Now, $y_3 - 13y_1 = (64e^{4x} - 2e^{-x}) - 13(4e^{4x} - 2e^{-x})$

$= 12e^{4x} + 24e^{-x} = 12(e^{4x} + 2e^{-x}) = 12y$

$$\therefore \quad \frac{y_3 - 13y_1}{y} = 12 \ \Rightarrow\ K = 3$$

27. (4): If the tangent $y = mx + \frac{1}{m}$ passes through $P(x_1, y_1)$, then $m^2x_1 - my_1 + 1 = 0$

If the roots are m_1 and m_2, then

$$m_1 + m_2 = \frac{y_1}{x_1},\ m_1m_2 = \frac{1}{x_1} \qquad ...(i)$$

The tangents are

$y - y_1 = m_1(x - x_1)$, $y - y_1 = m_2(x - x_1)$, $x = 0$

The vertices of the triangle are (x_1, y_1), $(0, y_1 - m_1x_1)$, $(0, y_1 - m_2x_1)$

$$\text{Area} = 1 \ \Rightarrow\ \begin{vmatrix} x_1 & y_1 & 1 \\ 0 & y_1 - m_1x_1 & 1 \\ 0 & y_1 - m_2x_1 & 1 \end{vmatrix} = \pm 2$$

$\Rightarrow \quad x_1(m_2 - m_1)x_1 = \pm 2$

Thus locus of P is $x^2(m_2 - m_1) = \pm 2$

$\Rightarrow \quad x^4[(m_1 + m_2)^2 - 4m_1m_2] = 4$

$\Rightarrow \quad x^2(y^2 - 4x) = 4$ [by (i)]

$\therefore \quad \lambda = 4$

28. (6): Let $P = (2 + s, 1 - 2s, -1 + s)$

and $Q = \left(\frac{8}{3} + 2t, -3 - t, 1 + t\right)$

Now, O, P, Q are collinear.

$$\therefore \quad \frac{2+s}{\frac{8}{3}+2t} = \frac{1-2s}{-3-t} = \frac{-1+s}{1+t}$$

On solving, we get $s = 3,\ t = \frac{1}{3}$

$$\Rightarrow \quad P = (5, -5, 2),\ Q = \left(\frac{10}{3}, -\frac{10}{3}, \frac{4}{3}\right)$$

$$d^2 = (PQ)^2 = \frac{25}{9} + \frac{25}{9} + \frac{4}{9} = 6.$$

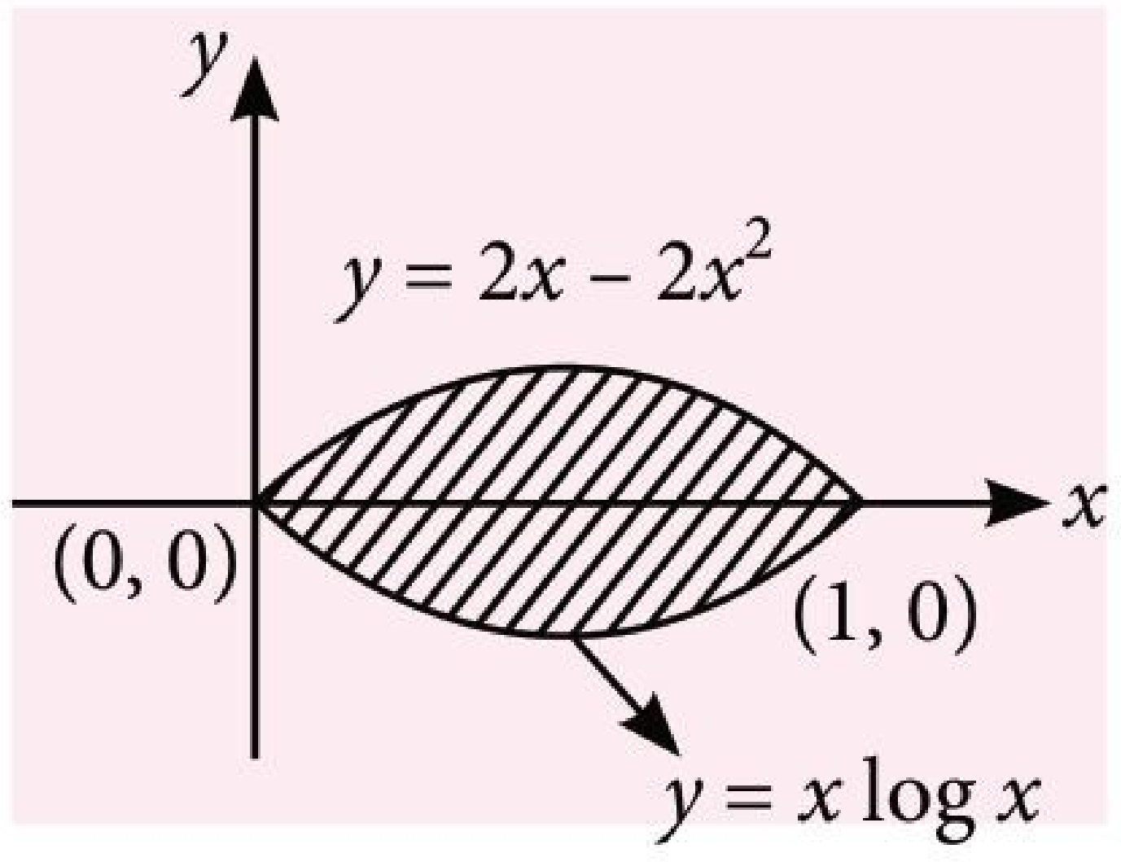
29. (0.58) : We have, $\frac{y}{x} = \log x$ and $y = -2x^2 + 2x$

$\Rightarrow \quad y = x\log x$ and $y = -2x^2 + 2x$

$y = 0 \ \Rightarrow\ \log x = 0$

$\Rightarrow \quad x = e^0 = 1 \ \Rightarrow\ x = 1, y = 0 \qquad ...(*)$

Now, $x = 1$, $y = 0$ satisfies both the curve means the point of intersection of these curves is given by (*).



$$\therefore \quad \text{Required area} = \int_0^1 \left[\left(2x - 2x^2\right) - x\log x\right]dx$$

$$= \left[x^2 - \frac{2}{3}x^3 - \frac{x^2}{2}\log x + \frac{x^2}{4}\right]_0^1$$

$$= \left(1 - \frac{2}{3} - \frac{1}{2}\log 1 + \frac{1}{4}\right) - \frac{1}{2}\operatorname*{Lt}_{x\to 0} x^2\log x$$

$$= \left(\frac{5}{4} - \frac{2}{3}\right) - \frac{1}{2}\operatorname*{Lt}_{x\to 0}\frac{\log x}{1/x^2} \quad (\infty/\infty \text{ form})$$

$$= \frac{7}{12} = 0.58 \text{ sq. units}$$

30. (2): At $P(x, y)$ for minima,

$f'(x) = 0,\ f''(x) > 0$

$\Rightarrow \quad x^2 + f^2(x) - 6 > 0 \ \Rightarrow\ x^2 + y^2 > 6$

(*i.e.*, P lies outside $x^2 + y^2 = 6$)

$\therefore \quad$ No. of tangents to $x^2 + y^2 = 4$ is 2.

JEE 2022 PRACTICE PAPER ADVANCED

Time : 1 hr. **Series - 1**

Topics to be covered : Complex Numbers & Quadratic Equations, Sequences & Series, Logarithms & their properties and Permutations & Combinations

PAPER-1

SECTION 1

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options. (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*

1. Find all complex numbers z which satisfy $|z|^2 - 2iz + 2c(1 + i) = 0$, where c is real.

(a) $z = c + i\left(-1 \pm \sqrt{1 - 2c - c^2}\right)$

(b) $z = c - i\left(-1 \pm \sqrt{1 + 2c + c^2}\right)$

(c) $z = c + i\left(1 \pm \sqrt{1 + 2c + c^2}\right)$

(d) $z = c - i\left(1 \pm \sqrt{1 - 2c - c^2}\right)$

2. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are non-zero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

(a) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$

(b) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

(c) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$

(d) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

3. Let A.P. $(a; d)$ denote the set of all the terms of an infinite arithmetic progression with first term a and common difference $d > 0$. If

A.P. $(1; 3) \cap$ A.P. $(2; 5) \cap$ A.P. $(3; 7)$ = A.P. $(a; d)$, then $a + d$ equals

(a) 150 (b) 157

(c) 153 (d) 263

4. Suppose, $\det\begin{bmatrix} \sum_{k=0}^{n} k & \sum_{k=0}^{n} {}^nC_k k^2 \\ \sum_{k=0}^{n} {}^nC_k k & \sum_{k=0}^{n} {}^nC_k 3^k \end{bmatrix} = 0$ holds for some positive integer n. Then $\sum_{k=0}^{n} \frac{{}^nC_k}{k+1}$ equals

(a) $\frac{31}{5}$ (b) $\frac{21}{5}$ (c) $\frac{41}{5}$ (d) $\frac{11}{5}$

SECTION 2

- *This section contains THREE (03) questions stems.*
- *There are TWO (02) questions corresponding to each question stem.*
- *The answer to each question is a NUMERICAL VALUE.*

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Question Stem for Question Nos. 5 and 6

Suppose $z = x + iy$ is identified with the matrix $\begin{bmatrix} x & y \\ -y & x \end{bmatrix}$.

5. The symmetric of given matrix means in terms of a complex number z is, Im(z) = _________.

6. The skew-symmetric of given matrix means in terms of a complex number z is, Re(z) = _________.

Question Stem for Question Nos. 7 and 8

Let a_n denote the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n = the number of such n-digit integers ending with digit 0.

7. If $a_{17} = a_\lambda + a_\mu$, then $\lambda - \mu$ equals _________.

8. The value of b_6 is _________.

Question Stem for Question Nos. 9 and 10

Let $S(n)$ denote the number of ordered pairs (x, y) satisfying $\frac{1}{x}+\frac{1}{y}=\frac{1}{n}$, $x, y, n \in N$.

9. The value of $S(6)$ = _________.

10. The value of $S(6) + S(7)$ = _________.

SECTION 3

- ***This section contains SIX (06) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).***

11. If $\binom{n}{r+1}=56$, $\binom{n}{r}=28$, $\binom{n}{r-1}=8$, then

(a) $r = 3$ (b) $r = 2$ (c) $n = 8$ (d) $n = 9$

12. If $a_1, a_2, ..., a_{50}$ are in G.P., then the value of $\frac{a_1-a_3+a_5-...+a_{49}}{a_2-a_4+a_6-...+a_{50}}$ is equal to

(a) 0 (b) 1 (c) $\frac{a_1}{a_2}$ (d) $\frac{a_{24}}{a_{25}}$

13. Let $S_n=\sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}}k^2$. Then S_n can take value(s)

(a) 1056 (b) 1088
(c) 1120 (d) 1332

14. If $3^x = 4^{x-1}$, then x =

(a) $\frac{2\log_3 2}{2\log_3 2-1}$ (b) $\frac{2}{2-\log_2 3}$

(c) $\frac{1}{1-\log_4 3}$ (d) $\frac{2\log_2 3}{2\log_2 3-1}$

15. Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S?

(a) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$ (b) $\left(-\frac{1}{\sqrt{5}}, 0\right)$

(c) $\left(0, \frac{1}{\sqrt{5}}\right)$ (d) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

16. Let a, b, x and y be real numbers such that $a - b = 1$ and $y \neq 0$. If the complex number $z = x + iy$ satisfies $\text{Im}\left(\frac{az+b}{z+1}\right)=y$, then which of the following is (are) possible value (s) of x?

(a) $1+\sqrt{1+y^2}$ (b) $-1+\sqrt{1-y^2}$

(c) $1-\sqrt{1+y^2}$ (d) $-1-\sqrt{1-y^2}$

SECTION 4

- ***This section contains THREE (03) questions.***
- ***The answer to each question is a NON-NEGATIVE INTEGER.***

17. If the equation $z^2-(3+i)z+m+2i=0$, $m \in R$ has a real root, then the value of m is _________.

18. The number of triplets (x, y, z) of positive integers, satisfying $2^x + 2^y + 2^z = 2336$ is _________.

19. Let $\omega \neq 1$ be a cube root of unity. Then the minimum of the set $\{|a + b\omega + c\omega^2|^2 : a, b, c$ distinct non-zero integers$\}$ equals _________.

PAPER-2

SECTION 1

- ***This section contains SIX (06) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).***

1. If z_1, z_2, z_3 and z_4 are the vertices of a square in that order, then

(a) $z_1 + z_3 = z_2 + z_4$

(b) $|z_1-z_2|=|z_2-z_3|=|z_3-z_4|=|z_4-z_1|$

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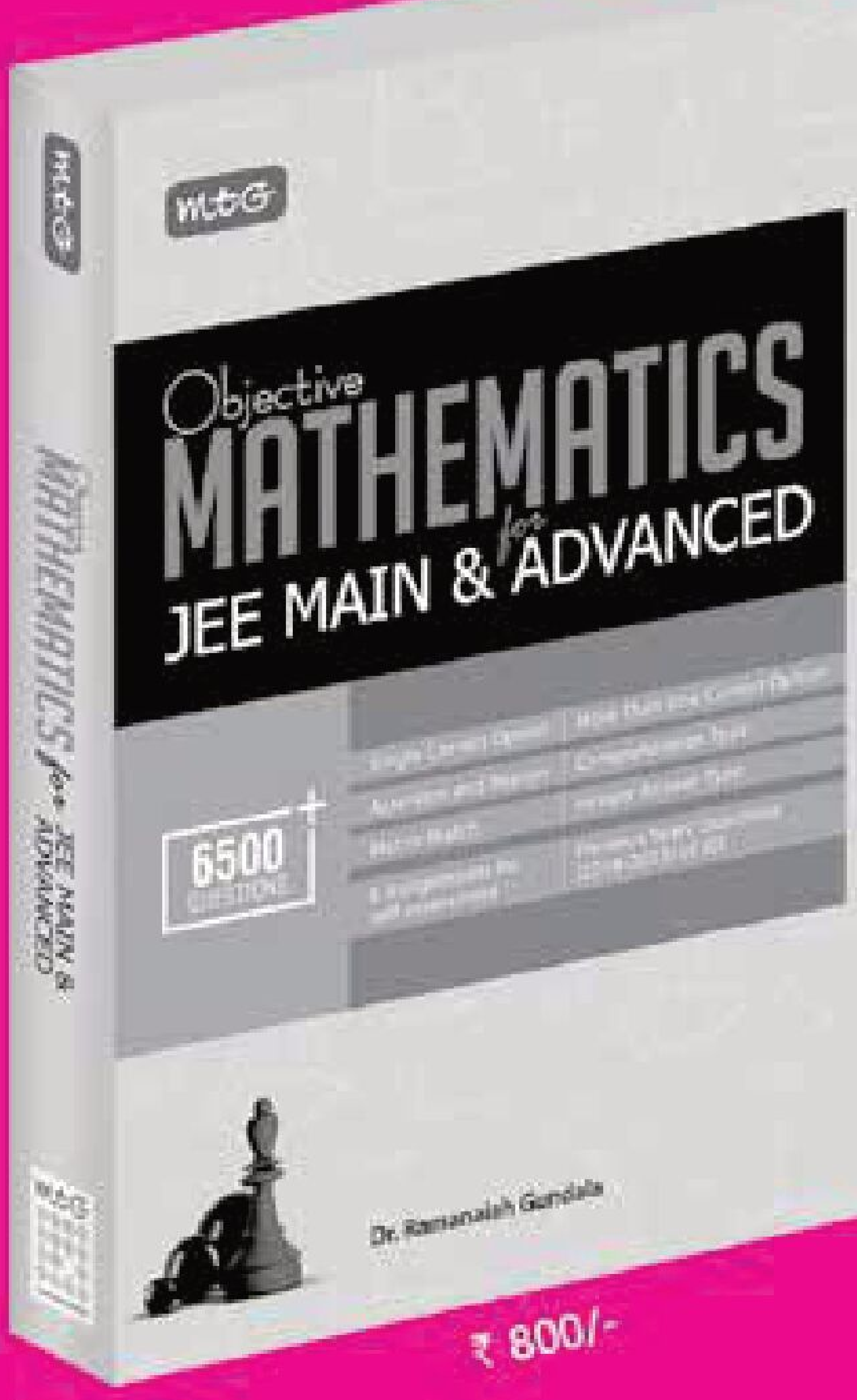

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(c) $|z_1 - z_3| = |z_2 - z_4|$

(d) $\dfrac{z_1 - z_3}{z_2 - z_4}$ is purely imaginary

2. For non-negative integers s and r, let

$$\binom{s}{r} = \begin{cases} \dfrac{s!}{r!(s-r)!}, & \text{if } r \le s \\ 0 &, \text{ if } r > s. \end{cases}$$

For positive integers m and n, let

$$g(m, n) = \sum_{p=0}^{m+n} \frac{f(m, n, p)}{\binom{n+p}{p}}$$

where for any non-negative integer p,

$$f(m, n, p) = \sum_{i=0}^{p} \binom{m}{i}\binom{n+i}{p}\binom{p+n}{p-i}.$$

Then which of the following statements is/are TRUE?

(a) $g(m, n) = g(n, m)$ for all positive integers m, n

(b) $g(m, n+1) = g(m+1, n)$ for all positive integers m, n

(c) $g(2m, 2n) = 2g(m, n)$ for all positive integers m, n

(d) $g(2m, 2n) = (g(m, n))^2$ for all positive integers m, n

3. Let $a_1, a_2, a_3, \ldots, a_n$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i$, $1 \le p \le 100$. For any integer n with $1 \le n \le 20$, let $m = 5n$. If $\dfrac{S_m}{S_n}$ does not depend on n, then a_2 is

(a) 0 (b) 3 (c) 6 (d) 9

4. If a, b, c are distinct positive numbers different from 1 such that $(\log_b a \log_c a - \log_a a) + (\log_c b \cdot \log_a b - \log_b b) + (\log_a c \log_b c - \log_c c) = 0$, then

(a) $\log a + \log b + \log c = 0$

(b) $abc = 1$

(c) $abc = 0$ (d) $\log a - \log b - \log c = 0$

5. Let $b_i > 1$ for $i = 1, 2, \ldots, 101$. Suppose $\log_e b_1, \log_e b_2, \ldots, \log_e b_{101}$ are in arithmetic progression (A.P.) with the common difference $\log_e 2$. Suppose $a_1, a_2, \ldots, a_{101}$ are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + \ldots + b_{51}$ and $s = a_1 + a_2 + \ldots + a_{51}$, then

(a) $s > t$ (b) $a_{101} < b_{101}$

(c) $s < t$ (d) $a_{101} > b_{101}$

6. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1 + x)^2 + (1 + x)^3 + \ldots + (1 + x)^{49} + (1 + mx)^{50}$ is $(3n + 1)\,{}^{51}C_3$ for some positive integer n. Then

(a) $m = 14$ (b) $m = 16$ (c) $n = 5$ (d) $n = 7$

SECTION 2

- *This section contains THREE (03) question stems.*
- *There are TWO (02) questions corresponding to each question stem.*
- *The answer to each question is a NUMERICAL VALUE.*

Question Stem for Question Nos. 7 and 8

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \ne \beta$. For $n = 0, 1, 2, \ldots$, let $a_n = p\alpha^n + q\beta^n$.

FACT : If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$.

7. If $a_{12} = a_r + a_s$, then $r \times s$ equals _________.

8. If $a_4 = 28$, then $p + 2q =$ _________.

Question Stem for Question Nos. 9 and 10

An infinite A.G.P. has its first five terms as : $a, -4, -3, 4, b$.

9. The absolute value of the fifth term b is _________.

10. If the sum of the infinite A.G.P is S, then $9S =$ _________.

Question Stem for Question Nos. 11 and 12

Let A, B, C, D, E be the smallest positive integers having 10, 12, 15, 16, 20 positive divisors respectively. Then

11. The value of $A + B =$ _________.

12. The value of $C + D =$ _________.

SECTION 3

- *This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO(02) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*
- *For each question, choose the option corresponding to the correct answer.*

Paragraph for Q. 13-14

Suppose the complex number $z = x + iy$ be defined by the vector $\vec{z} = x\hat{i} + y\hat{j}$.

13. Which of the following is wrong?

(a) $z_1 + z_2$ is the vertex of the parallelogram opposite to origin, whose two diagonally opposite vertices are given by z_1 and z_2.

(b) $z_1 = z_2$ if and only if $\text{Re}(z_1) = \text{Re}(z_2)$ and $\text{Im}(z_1) = \text{Im}(z_2)$.

(c) $\dfrac{1}{z}$ is defined for every $z \in C - \{0\}$.

(d) None of these

14. The dot product of the vectors $\vec{z}_1$ and $\vec{z}_2$, is equal to

(a) $\text{Im}(z_1\bar{z}_2)$ (b) $\text{Re}(z_1\bar{z}_2)$

(c) $\text{Im}(\bar{z}_1 z_2)$ (d) $\text{Re}(z_1 z_2)$

Paragraph for Q. 15-16

Consider the set $A = \{0, 1, 2, 3, 4, 5\}$.

15. The number of 6-digit numbers formed , using the digits in A without repetition of digits is

(a) 540 (b) 640
(c) 720 (d) 600

16. Three digit numbers are formed using the digits in A without repetition of digits. The number of these numbers in which the digits either increase or decrease is

(a) 50 (b) 45
(c) 30 (d) 24

SECTION 4

- *This section contains THREE (03) questions.*
- *The answer to each question is a NON-NEGATIVE INTEGER.*

17. In a hotel, four rooms are available. Six persons are to be accommodated in these four rooms in such a way that each of these rooms contains at least one person and at most two persons. Then the number of all possible ways in which this can be done is _____.

18. Let m be the minimum possible value of $\log_3(3^{y_1} + 3^{y_2} + 3^{y_3})$, where y_1, y_2, y_3 are real numbers for which $y_1 + y_2 + y_3 = 9$. Let M be the maximum possible value of $(\log_3 x_1 + \log_3 x_2 + \log_3 x_3)$, where x_1, x_2, x_3 are positive real numbers for which $x_1 + x_2 + x_3 = 9$. Then the value of $\log_2(m^3) + \log_3(M^2)$ is _____.

19. For $r = 0, 1, ..., 10$, let A_r, B_r and C_r denote respectively, the coefficients of x^r in the expansions of $(1 + x)^{10}, (1 + x)^{20}$ and $(1 + x)^{30}$. If $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ $= C_p - B_q$, then value of $p + q$ which is less than 25 is equal to _________.

SOLUTIONS

PAPER-1

1. (a) : Put $z = a + ib$, then $a^2 + b^2 - 2ai + 2b + 2c + 2ci = 0$

$\Rightarrow \quad (a^2 + b^2 + 2b + 2c) + (2c - 2a)i = 0$

$\Rightarrow \quad a^2 + b^2 + 2b + 2c = 0$ and $2c - 2a = 0 \Rightarrow a = c$

Now, $b^2 + 2b + (c^2 + 2c) = 0$

$\Rightarrow \quad b = -1 \pm \sqrt{1-2c-c^2}$

Since b is real, so, $1-2c-c^2 \geq 0$

$\Rightarrow \quad c \in [-1-\sqrt{2},\ -1+\sqrt{2}]$

$\Rightarrow \quad z = c + i(-1 \pm \sqrt{1-2c-c^2})$

2. (b) : $\alpha + \beta = -p$

$\alpha^3 + \beta^3 = q$

Since, $(\alpha + \beta)^3 = \alpha^3 + \beta^3 + 3\alpha\beta(\alpha + \beta)$

$\Rightarrow \quad -p^3 = q - 3\alpha\beta p$

$$\Rightarrow \alpha\beta = \frac{p^3+q}{3p}$$

$$\text{Now, } \frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} = \frac{(\alpha+\beta)^2}{\alpha\beta}-2$$

$$= \frac{(p^2)(3p)}{p^3+q}-2 = \frac{3p^3-2p^3-2q}{p^3+q} = \frac{p^3-2q}{p^3+q}$$

Thus required equation is $x^2 - \left(\frac{p^3-2q}{p^3+q}\right)x + 1 = 0$

$\Rightarrow \quad (p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

3. (b) : Let $1 + 3m = 2 + 5n = 3 + 7r$

The first common term is given by setting $r = 7$

$\therefore \quad a = 52$

and d = least common multiple of 3, 5, 7 = 105

Hence, $a + d = 105 + 52 = 157$

4. (a) : Given, $\det\begin{bmatrix} \sum_{k=0}^{n} k & \sum_{k=0}^{n} {}^nC_k k^2 \\ \sum_{k=0}^{n} {}^nC_k k & \sum_{k=0}^{n} {}^nC_k 3^k \end{bmatrix} = 0$

$$\Rightarrow \begin{vmatrix} \frac{n(n+1)}{2} & n(n+1)2^{n-2} \\ n\,2^{n-1} & 4^n \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 2^{n-1} \\ n\cdot 2^{n-1} & 2^{2n} \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 1 \\ n\cdot 2^{n-1} & 2^{n+1} \end{vmatrix} = 0 \Rightarrow n = 4$$

$$\text{Now, } \sum_{k=0}^{n} \frac{{}^nC_k}{k+1} = \sum_{k=0}^{4} \frac{{}^4C_k}{k+1} = \frac{1}{5}\sum_{k=0}^{4} {}^5C_{k+1}$$

$$= \frac{1}{5}[2^5-1] = \frac{31}{5}$$

5. (0) : If matrix is symmetric, then

$$\begin{bmatrix} x & y \\ -y & x \end{bmatrix} = \begin{bmatrix} x & -y \\ y & x \end{bmatrix}$$

$\Rightarrow \quad y = 0 \Rightarrow \text{Im}(z) = 0.$

6. (0) : If matrix is skew symmetric, then

$$\begin{bmatrix} x & y \\ -y & x \end{bmatrix} = \begin{bmatrix} -x & y \\ -y & -x \end{bmatrix}$$

$\Rightarrow \quad x = 0 \Rightarrow \text{Re}(z) = 0.$

7. (1) : We can setup a recurrence relation for the problem.

As consecutive digits can't be both zero, we have $a_n = a_{n-1} + a_{n-2}$. Also $b_n = a_{n-1}$, $c_n = a_{n-2}$

We have $a_1 = 1$, $a_2 = 2$ as starting values, then the other values are $a_3 = 3$, $a_4 = 5$, $a_5 = 8$, $a_6 = 13$

We have $a_{17} = a_{16} + a_{15}$. Note that it is actually a Fibonacci sequence in disguise.

8. (8) : As $b_n = a_{n-1}$

we have $b_6 = a_5$ and $a_5 = 8$ so $b_6 = 8$

9. (9) : The number of pairs satisfying $\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$ is equal to the number of divisors of $6^2 = 2^2 \cdot 3^2$

$\therefore \quad S(6) = 3 \cdot 3 = 9$

10. (12) : $S(6) = 9$

$S(7)$ is the number of divisors of 7^2

$\therefore \quad S(7) = 3$

Thus, $S(6) + S(7) = 12$

11. (b, c) : $\binom{n}{r+1} \div \binom{n}{r} = \frac{n-r}{r+1} = \frac{56}{28} = 2$

$\Rightarrow \quad n = 3r + 2 \qquad \text{... (i)}$

$\binom{n}{r} \div \binom{n}{r-1} = \frac{n-r+1}{r} = \frac{28}{8} = \frac{7}{2}$

$\Rightarrow \quad 2n = 9r - 2 \qquad \text{... (ii)}$

From (i) and (ii), we get $r = 2$, $n = 8$

12. (c, d) : Since $a_1, a_2, a_3, ..., a_{50}$ are in G.P., we have $a_2 = ar$, $a_3 = ar^2$, $a_4 = ar^3$, ..., $a_{49} = ar^{48}$, $a_{50} = ar^{49}$; where a is first term and r is common ratio.

So, $\frac{a_1 - a_3 + a_5 - ... + a_{49}}{a_2 - a_4 + a_6 - ... + a_{50}} = \frac{a - ar^2 + ar^4 - ... + ar^{48}}{ar - ar^3 + ar^5 - ... + ar^{49}}$

$= \frac{a(1 - r^2 + r^4 - ... + r^{48})}{ar(1 - r^2 + r^4 - ... + r^{48})} = \frac{1}{r}$

Also, $\frac{a_1}{a_2} = \frac{a}{ar} = \frac{1}{r}$

$\therefore \quad \frac{a_1 - a_3 + a_5 - ... + a_{49}}{a_2 - a_4 + a_6 - ... + a_{50}} = \frac{a_1}{a_2}$

13. (a, d) : Consider $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$

$= -1^2 - 2^2 + 3^2 + 4^2 - 5^2 - 6^2 + 7^2 + 8^2 - .....$

$= (3^2 - 1^2) + (4^2 - 2^2) + ...$

$= 2(4 + 6 + 12 + 14 + ...)$ to $2n$ terms

$= 2[(4 + 12 + 20 + ...) + (6 + 14 + 22 + ...)]$

$= 2\left[\frac{n}{2}(8 + 8(n-1)) + \frac{n}{2}(12 + 8(n-1))\right]$

$= 2(4n^2 + 4n^2 + 2n) = 2(8n^2 + 2n) = 4n(4n + 1)$

As $1056 = 32 \times 33$, $1088 = 32 \times 34$,

$1120 = 32 \times 35$, $1332 = 36 \times 37$

we have only 1056 and 1332 as the values that S_n can take.

14. (a, b, c) : Given $3^x = 4^{x-1}$

Taking log to base 4, we get

$x\log_4 3 = x - 1$

$\Rightarrow \quad x\{1 - \log_4 3\} = 1$

$\therefore \quad x = \frac{1}{1 - \log_4 3} = \frac{1}{1 - \frac{1}{2}\log_2 3} = \frac{2}{2 - \log_2 3}$

$= \frac{2}{2 - \frac{1}{\log_3 2}} = \frac{2\log_3 2}{2\log_3 2 - 1}$

15. (a, d) : We have $\alpha x^2 - x + \alpha = 0$; $D = 1 - 4\alpha^2$

For distinct real roots, $1 - 4\alpha^2 > 0$

i.e., $\alpha \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

Now, $|x_1 - x_2| < 1$ *i.e.*, $(x_1 - x_2)^2 < 1$

$\Rightarrow \quad 1 - 4\alpha^2 < \alpha^2$

$\Rightarrow \quad \alpha \in \left(-\infty, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

Combining the two bounds, we have

$\alpha \in \left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

$\therefore \quad S = \left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

16. (b, d) : Consider $\text{Im}\left(\frac{a(x + iy) + b}{x + iy + 1}\right) = y$

$\Rightarrow \quad \text{Im}\left(\frac{(ax + b) + iay}{x + 1 + iy}\right) = y$

$\Rightarrow \quad \text{Im}\left(\frac{[(ax + b) + iay][(x + 1) - iy]}{(x + 1)^2 + y^2}\right) = y$

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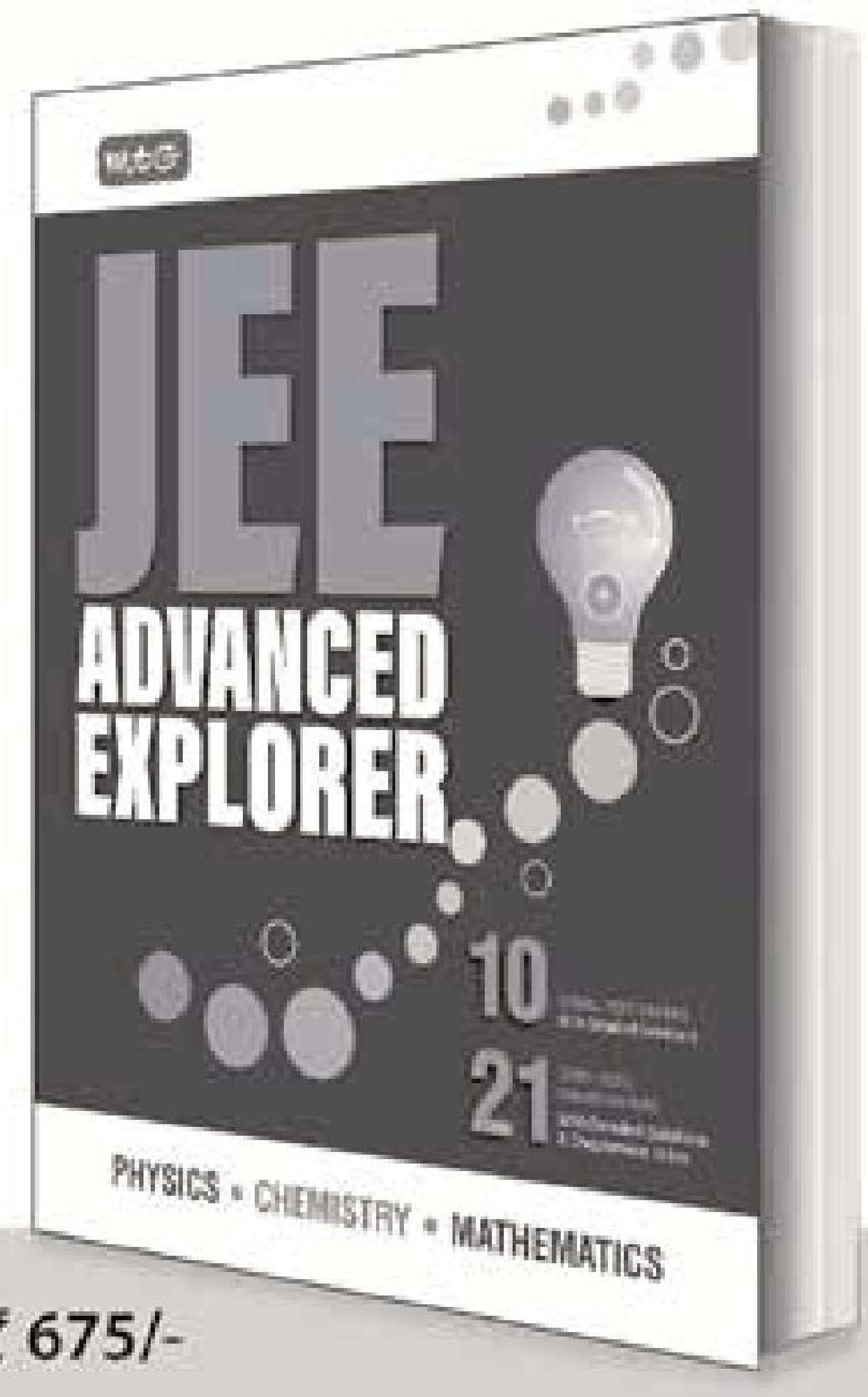
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$$\Rightarrow \quad \operatorname{Im}\left(\frac{(ax+b)(x+1)+ay^2}{(x+1)^2+y^2}+i\frac{ay(x+1)-y(ax+b)}{(x+1)^2+y^2}\right)=y$$

We then have,

$$\frac{ay-by}{(x+1)^2+y^2}=y \;\Rightarrow\; y=y\{(x+1)^2+y^2\}$$

$$[\because a-b=1(\text{given})]$$

As $y \neq 0$, we have $(x + 1)^2 + y^2 = 1$

$$\Rightarrow \quad x+1=\pm\sqrt{1-y^2}$$

$$\therefore \quad x=-1+\sqrt{1-y^2} \quad \text{or} \quad -1-\sqrt{1-y^2}$$

17. (2) : Let r be the real root.

Then, $r^2-(3+i)r+m+2i=0$.

Now, equating the real and imaginary parts, we get $r^2 - 3r + m = 0$ and $2 - r = 0$

$\Rightarrow \quad r = 2$

$\therefore \quad m = 6 - 4 = 2$

18. (6) : $2^x + 2^y + 2^z = 2336 = 2^5 \times 73 = 2^5 \times (1 + 8 + 64)$

$= 2^5(2^0 + 2^3 + 2^6) = 2^5 + 2^8 + 2^{11}$

Number of triplets consisting of 5, 8 and 11 is 3! = 6

19. (3) : $|a + b\omega + c\omega^2|^2 = (a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$

$= a^2 + b^2 + c^2 - ab - bc - ca$

$$=\frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2] \;=\frac{1}{2}(1+1+4)=3$$

Since it will be minimum only if a, b, c are consecutive integers.

PAPER-2

1. (a, b, c, d) : Let the four points represented by z_1, z_2, z_3 and z_4 be A, B, C and D respectively. Since $ABCD$ is a square, the mid point of AC = the mid point of BD.

$$\Rightarrow \quad \frac{1}{2}(z_1+z_3)=\frac{1}{2}(z_2+z_4)$$

$$\Rightarrow \quad z_1+z_3=z_2+z_4$$

Also, $AB = BC = CD = DA$

$$\Rightarrow \; |z_1-z_2| = |z_2-z_3| = |z_3-z_4| = |z_4-z_1|$$

Since diagonals of the square $ABCD$ are equal.

$\therefore \quad AC = BD$

$$\Rightarrow \quad |z_1-z_3|=|z_2-z_4|$$

Also, since $AC \perp BD$, or $\dfrac{z_1-z_3}{z_2-z_4}$ is purely imaginary.

2. (a, b, d) : We have,

$$f(m, n, p) = \sum_{i=0}^{p} {}^{m}C_i \cdot {}^{n+i}C_p \cdot {}^{p+n}C_{p-i}$$

$$=\sum_{i=0}^{p} {}^{m}C_i \cdot \frac{(n+i)!}{p!\;(n+i-p)!}\cdot\frac{(p+n)!}{(p-i)!\;(n+i)!}$$

$$=\sum_{i=0}^{p} {}^{m}C_i \cdot \frac{(p+n)!}{p!(n+i-p)!\;(p-i)!}$$

$$=\sum_{i=0}^{p} {}^{m}C_i \cdot \frac{(p+n)!}{p!(n)!}\cdot\frac{n!}{(n-p+i)!(p-i)!}$$

$$=\sum_{i=0}^{p} {}^{m}C_i \cdot {}^{n+p}C_p \cdot {}^{n}C_{p-i} \;= {}^{n+p}C_p \sum_{i=0}^{p} {}^{m}C_i \cdot {}^{n}C_{p-i}$$

$= {}^{n+p}C_p \cdot {}^{m+n}C_p$ (Using Vandermonde's identity)

Now, $g(m,n)=\displaystyle\sum_{p=0}^{m+n}\frac{f(m,n,p)}{{}^{n+p}C_p}=\sum_{p=0}^{m+n}{}^{m+n}C_p=2^{m+n}$

$g(m, n)$ is symmetric in m and n.

$\therefore \quad g(m, n) = g(n, m)$

Also, $g(m, n + 1) = 2^{m+n+1} = 2^{(m+1)+n} = g(m + 1, n)$

$g(2m, 2n) = 2^{2m+2n} = (2^{m+n})^2 = (g(m, n))^2$

3. (b, d) : $\dfrac{S_m}{S_n}=\dfrac{S_{5n}}{S_n}=\dfrac{\frac{5n}{2}(6+(5n-1)d)}{\frac{n}{2}(6+(n-1)d)}$

For $d = 6$ we have S_{5n}/S_n is independent of 'n'. Also, For $d = 0$, it's trivially true.

So, $a_2 = 9$ or 3.

4. (a, b) : If $\alpha = \log a$, $\beta = \log b$, $\gamma = \log c$, the given equation becomes

$$\frac{\alpha^2}{\beta\gamma}-1+\frac{\beta^2}{\gamma\alpha}-1+\frac{\gamma^2}{\alpha\beta}-1=0$$

$\Rightarrow \quad \alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma = 0 \;\Rightarrow\; \alpha + \beta + \gamma = 0$

$\Rightarrow \quad \log a + \log b + \log c = 0$

$\Rightarrow \quad \log(abc) = 0 \;\Rightarrow\; abc = 1.$

5. (a, b) : We know that $\ln b_2 - \ln b_1 = \ln_2$

$$\therefore \quad \frac{b_2}{b_1}=2$$

$\Rightarrow \quad b_1, b_2, ..., b_{101}$ are in G.P.

Let $a_1 = b_1 = \alpha$. $\Rightarrow \; t = \alpha(2^{51} - 1)$

and $s=\dfrac{51}{2}(2\alpha+50d)$ $\qquad$ [d = common ratio of A.P.]

Now, $a_{51} = b_{51}$

$\Rightarrow \quad \alpha + 50d = \alpha \cdot 2^{50}$

$\Rightarrow \quad 50d = \alpha(2^{50} - 1)$

$$\therefore \quad s=\alpha\cdot\frac{51}{2}(2^{50}+1)=\alpha\left(51\cdot 2^{49}+\frac{51}{2}\right)$$

As $s-t=\alpha\left(47\cdot 2^{49}+\dfrac{53}{2}\right)>0 \;\therefore\; s>t$

$a_{101} = \alpha + 2\alpha \cdot 2^{50} - 2\alpha = \alpha(2^{51} - 1)$

$b_{101} = \alpha \cdot 2^{100}$

$\therefore \quad b_{101} > a_{101}$

6. (b, c) : The coefficient of x^2 in the expansion of $(1 + x)^2 + (1 + x)^3 + ... + (1 + mx)^{50}$ is

${}^2C_2 + {}^3C_2 + ... + {}^{49}C_2 + {}^{50}C_2\, m^2 = {}^{50}C_3 + {}^{50}C_2\, m^2$

It is given to be $(3n + 1)\ {}^{51}C_3$

Now, $\dfrac{50 \cdot 49 \cdot 48}{6} + \dfrac{50 \cdot 49}{2} m^2 = (3n+1)\dfrac{51 \cdot 50 \cdot 49}{6}$

$\Rightarrow \quad 51n + 1 = m^2$

As $51n + 1$ must be perfect square.

So, we have by inspection, $n = 5$.

$\therefore \quad m = 16$

7. (110) : As α and β are the roots of the equation $x^2 - x - 1 = 0$, so we have,

$\alpha^2 = \alpha + 1$

$\beta^2 = \beta + 1$

Multiply by $p\alpha^n$ and $q\beta^n$ respectively, we have

$p\alpha^{n+2} = p\alpha^{n+1} + p\alpha^n$ ...(i)

$q\beta^{n+2} = q\beta^{n+1} + q\beta^n$ ...(ii)

Adding (i) and (ii), we get

$p\alpha^{n+2} + q\beta^{n+2} = (p\alpha^{n+1} + q\beta^{n+1}) + (p\alpha^n + q\beta^n)$

which becomes, $a_{n+2} = a_{n+1} + a_n,\ n \geq 0$

Setting $n = 10$, we have $a_{12} = a_{11} + a_{10} = a_r + a_s$

$\therefore \quad r \times s = 11 \times 10 = 110$

8. (12) : $a_4 = a_3 + a_2 = a_2 + a_1 + a_2$

$= 2a_2 + a_1 = 2(a_1 + a_0) + a_1$

$= 3a_1 + 2a_0 = 3p\alpha + 3q\beta + 2(p + q)$

$$= 3p\left(\frac{1+\sqrt{5}}{2}\right) + 3q\left(\frac{1-\sqrt{5}}{2}\right) + 2(p+q)$$

$$= \left(\frac{3p}{2} + \frac{3q}{2} + 2p + 2q\right) + \frac{3\sqrt{5}}{2}(p-q)$$

As $a_4 = 28$, we have $p = q$ and $\dfrac{3p}{2} + \dfrac{3q}{2} + 2(p+q) = 28$

The relation gives $7p = 28$

$\therefore \quad p = 4$

9. (3.25) : $(a + d)r = -4,\ (a + 2d)r^2 = -3,\ (a + 3d)r^3 = 4$

$$\Rightarrow a + d = -\frac{4}{r},\ a + 2d = -\frac{3}{r^2},\ a + 3d = \frac{4}{r^3}$$

$$\Rightarrow d = -\frac{3}{r^2} + \frac{4}{r},\ d = \frac{4}{r^3} + \frac{3}{r^2} \ \therefore\ -\frac{3}{r^2} + \frac{4}{r} = \frac{4}{r^3} + \frac{3}{r^2}$$

$\Rightarrow \quad 2r^2 - 3r - 2 = 0 \ \Rightarrow\ r = 2,\ -\dfrac{1}{2}$

For infinite A.G.P. $|r| < 1 \Rightarrow r = -\dfrac{1}{2}$

$$d = -\frac{3}{r^2} + \frac{4}{r} = -12 - 8 = -20,\ a = -d - \frac{4}{r} = 20 + 8 = 28.$$

$a = 28,\ d = -20,\ r = -\dfrac{1}{2}$

$\therefore \quad b = (a + 4d)r^4 = \dfrac{28 - 80}{16} = -\dfrac{13}{4}$

$\Rightarrow \quad |b| = 3.25$

10. (208) : $S = a + (a + d)r + (a + 2d)r^2 + ...$

$Sr = ar + (a + d)r^2 + ...$

$S - Sr = a + dr + dr^2 + dr^3 + ... = a + \dfrac{dr}{1-r}$

$$\therefore\ S = \frac{1}{1-r}\left[a + \frac{dr}{1-r}\right] = \frac{2}{3}\left[28 + 10 \times \frac{2}{3}\right] = \frac{208}{9}.$$

$\Rightarrow \quad 9S = 208$

11. (108) : $A + B = 48 + 60 = 108$

12. (354) : $C + D = 144 + 210 = 354$

13. (c) : We know that the rule of addition of complex numbers is the same as the rule of addition of vectors. Hence (a) is true. (b) is also true. (c) is wrong as reciprocal of vectors is not defined.

14. (b) : $\vec{z}_1 \cdot \vec{z}_2 = (x_1\hat{i} + y_1\hat{j}) \cdot (x_2\hat{i} + y_2\hat{j}) = x_1x_2 + y_1y_2$

$= \text{Re}(z_1\bar{z}_2) = \text{Re}(\bar{z}_1z_2)$

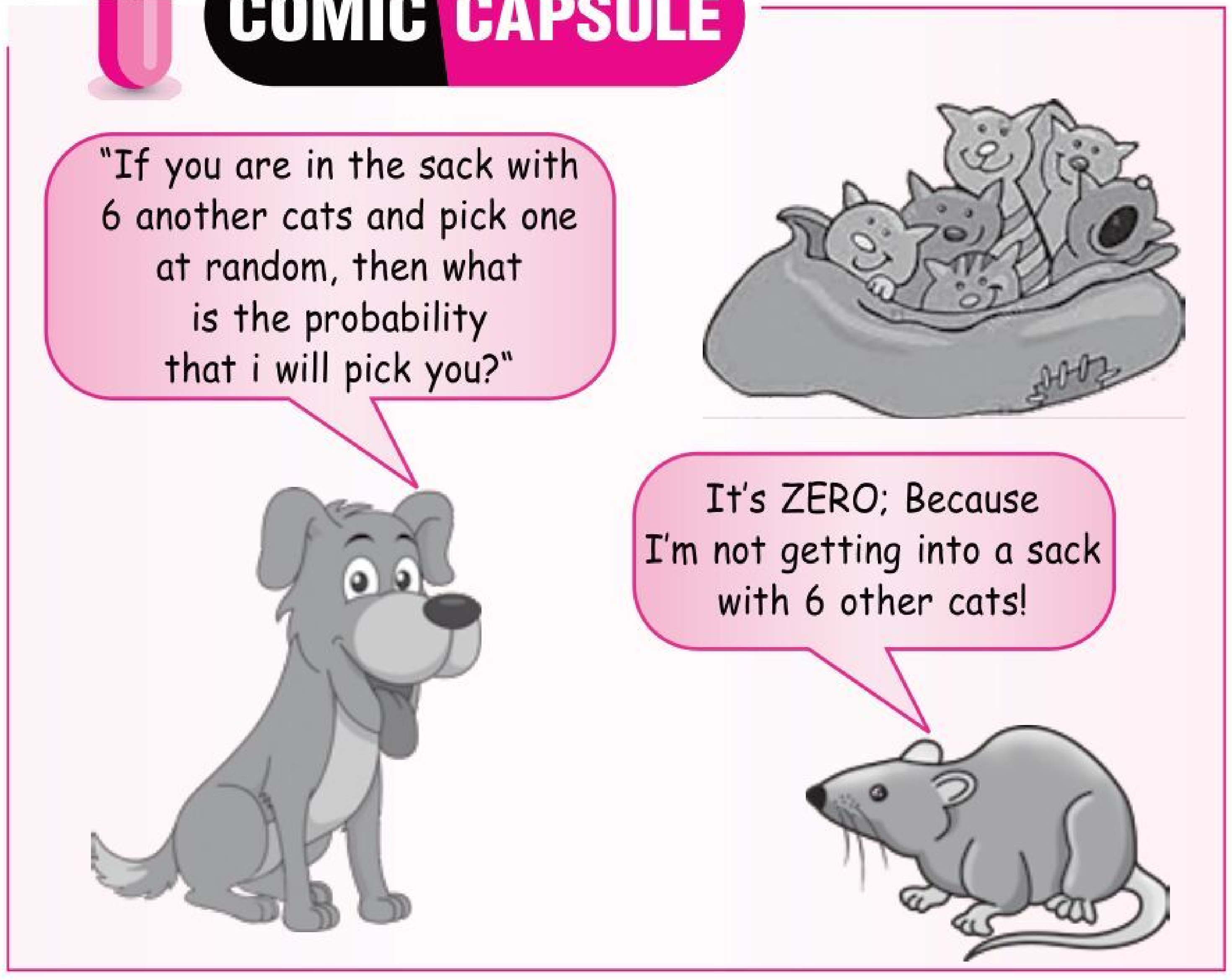
15. (d) : The desired number is $6! - 5! = 720 - 120 = 600$.

16. (c) : 3-digit numbers abc, $a < b < c$, $a \neq 0$

Their number is $\dbinom{5}{3} = \dbinom{5}{2} = 10$

$\Rightarrow \quad$ 3-digit numbers abc, $a > b > c$

COMIC CAPSULE


"If you are in the sack with 6 another cats and pick one at random, then what is the probability that i will pick you?"
It's ZERO; Because I'm not getting into a sack with 6 other cats!

Their number is $\binom{6}{3} = 20$

The desired number is 10 + 20 = 30.

17. (1080) : We can have 4 groups of person such that 2 groups contain 1 person each and other 2 groups contain 2 persons each.

i.e., 6 = 2 + 2 + 1 + 1

Number of ways to form the groups

$$= \frac{6!}{2!\,2!\,1!\,1!} \times \frac{1}{2!} \cdot \frac{1}{2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 2 \cdot 2 \cdot 2} = 45$$

They can be assigned 4 rooms in 4! ways.

So the number of ways = 45 × 4!

= 45 × 24 = 1080

18. (8) : Using A.M. ≥ G.M. inequality, we have

$$\frac{3^{y_1} + 3^{y_2} + 3^{y_3}}{3} \geq \sqrt[3]{3^{y_1} \cdot 3^{y_2} \cdot 3^{y_3}}$$

$$\Rightarrow \quad 3^{y_1} + 3^{y_2} + 3^{y_3} \geq 3 \cdot \sqrt[3]{3^{y_1 + y_2 + y_3}}$$

$$= 3 \cdot \sqrt[3]{3^9} = 3 \cdot 3^3 = 3^4$$

$$\Rightarrow \quad \log_3(3^{y_1} + 3^{y_2} + 3^{y_3}) \geq 4$$

$\therefore \quad m = 4.$

Again using A.M. ≥ G.M., we have

$$\frac{x_1 + x_2 + x_3}{3} \geq \sqrt[3]{x_1 x_2 x_3}$$

$\Rightarrow \quad x_1x_2x_3 \leq 27 = 3^3 \Rightarrow \log_3 x_1 + \log_3 x_2 + \log_3 x_3 \leq 3$

$\therefore \quad M = 3$

Now, $\log_2(m^3) + \log_3(M^2) = 3\log_2 m + 2\log_3 M$

$= 3\log_2 4 + 2\log_3 3 = 6 + 2 = 8$

19 (20) : The given expression

$$= \sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$$

$\sum_{r=1}^{10} A_r B_r =$ coefficient of x^{20} in the expansion of

$\{(1 + x)^{10}(x + 1)^{20}\} - 1 = C_{20} - 1 = C_{10} - 1$ and

$\sum_{r=1}^{10} (A_r)^2 =$ coefficient of x^{10} in $\{(1 + x)^{10}(x + 1)^{10}\} - 1$

$= B_{10} - 1$

We have the given expression

$= B_{10}(C_{10} - 1) - C_{10}(B_{10} - 1) = C_{10} - B_{10} = C_p - B_q$

$\therefore \quad p + q = 10 + 10 = 20$

or $\quad p + q = 20 + 10 = 30$

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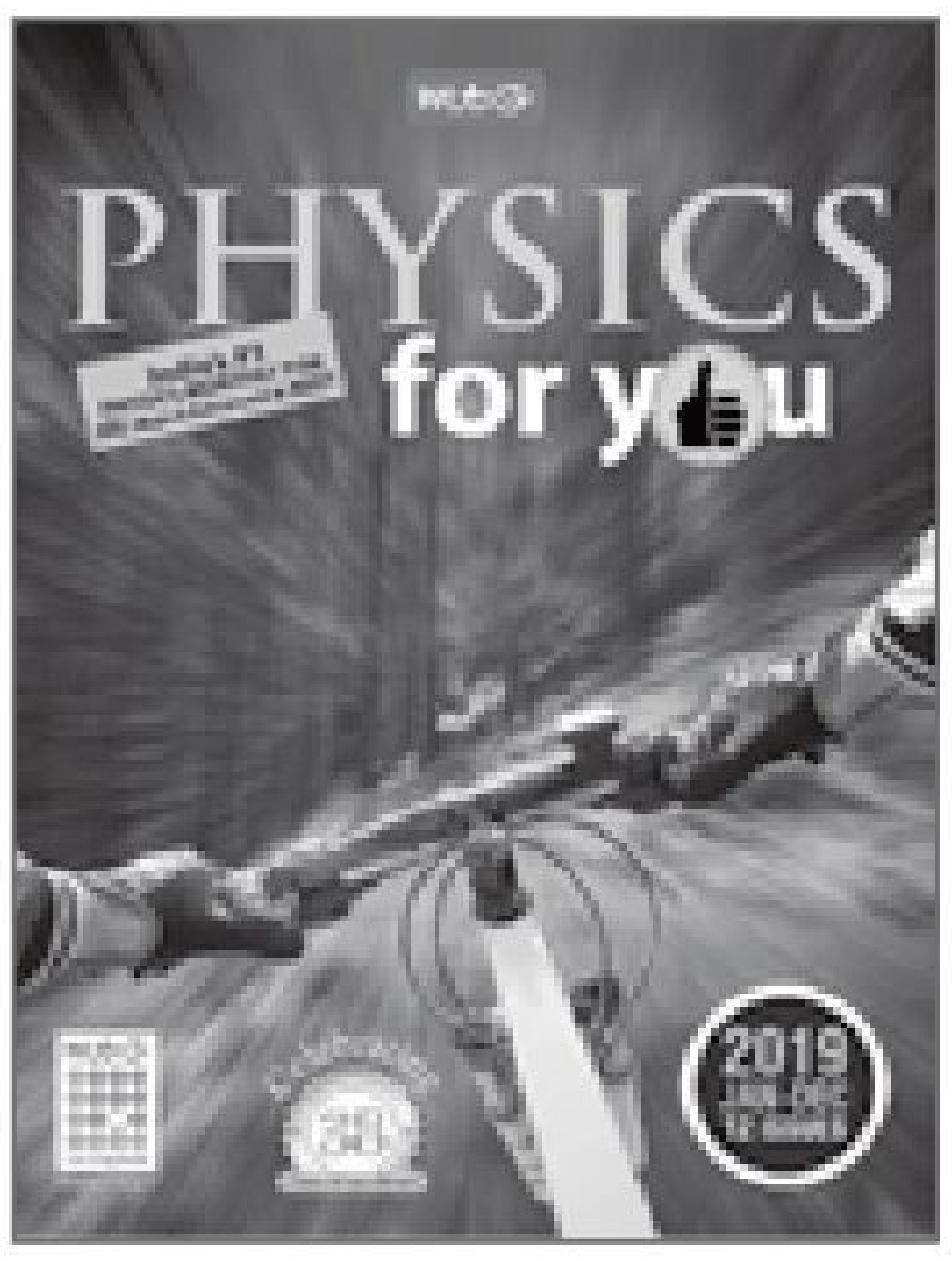

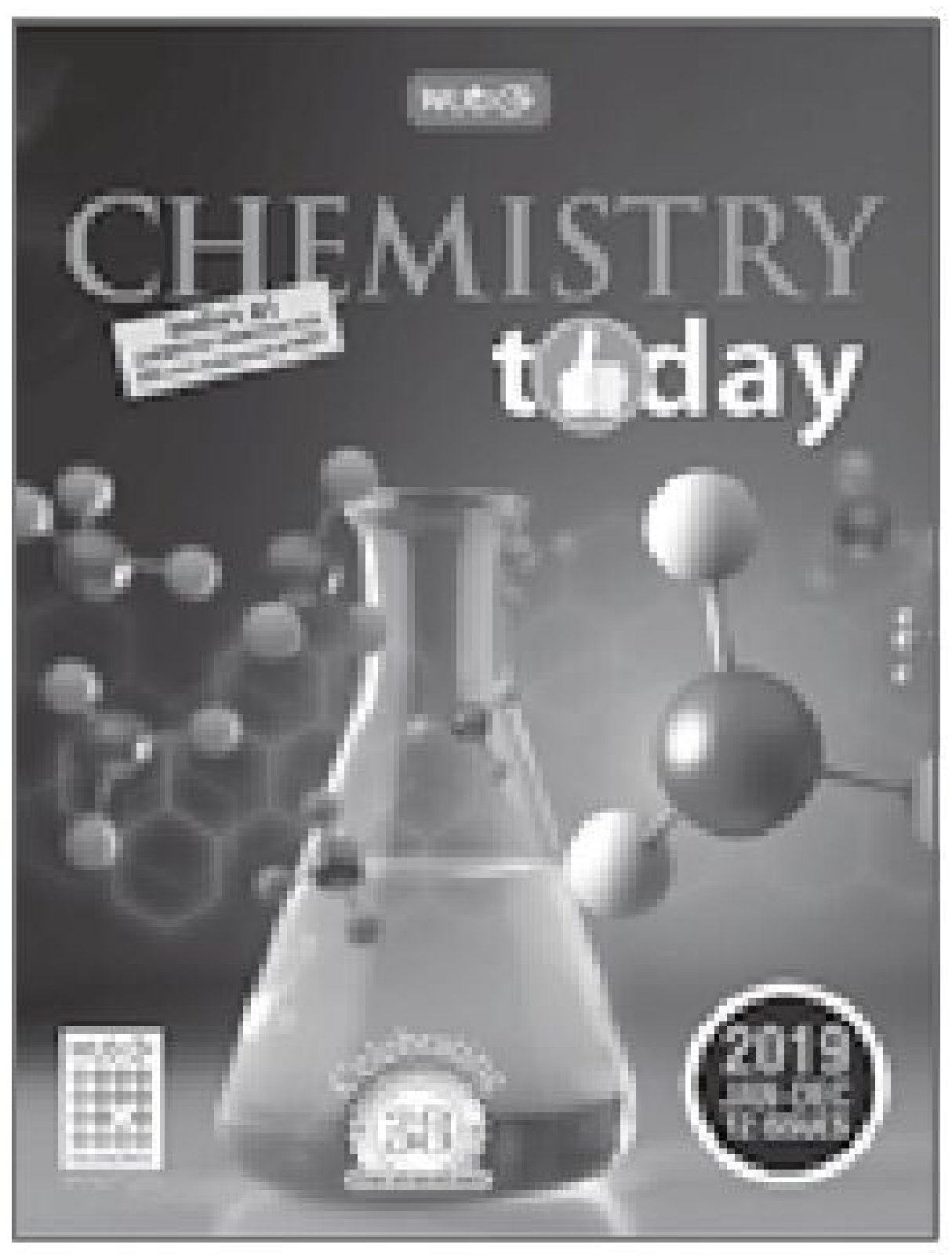

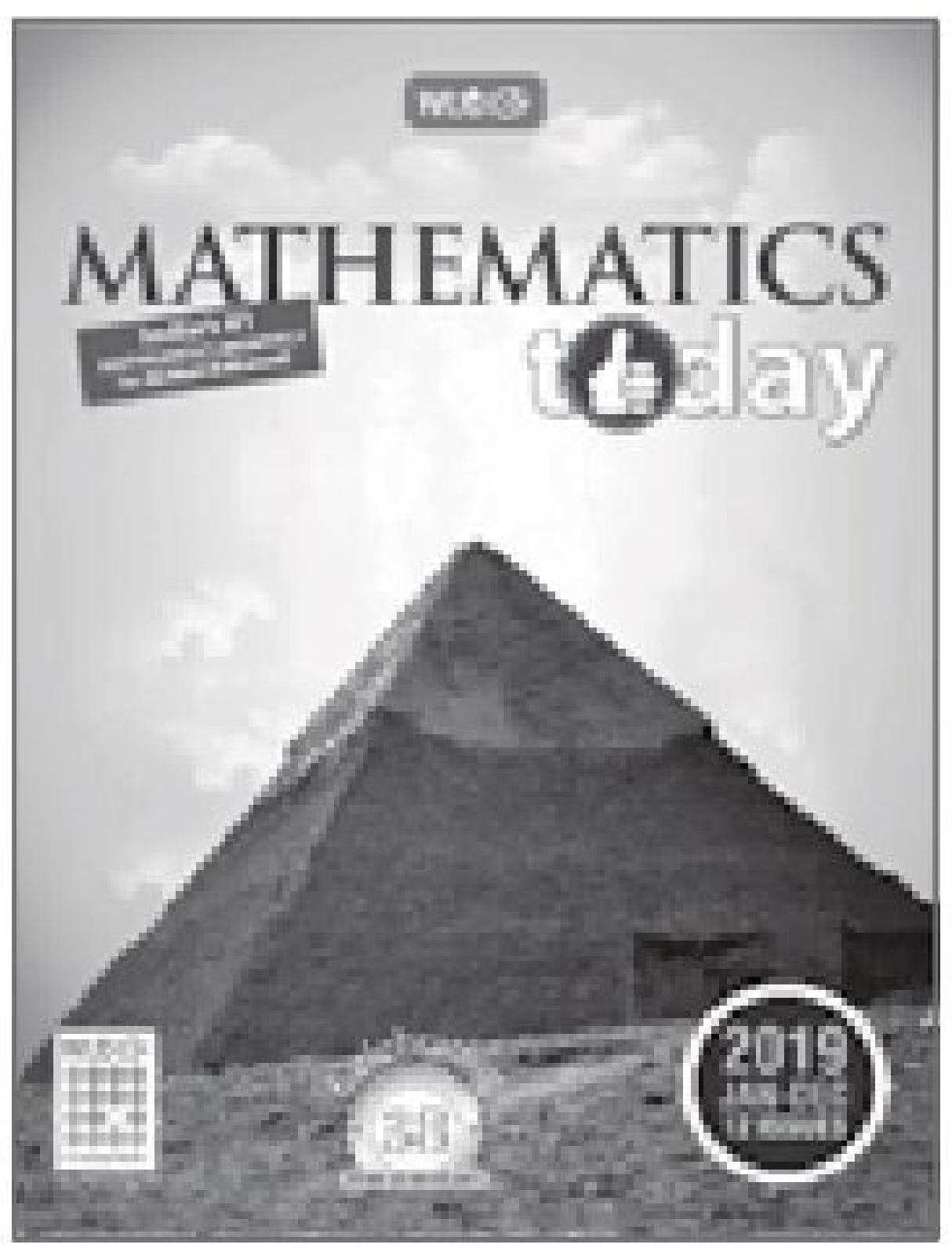

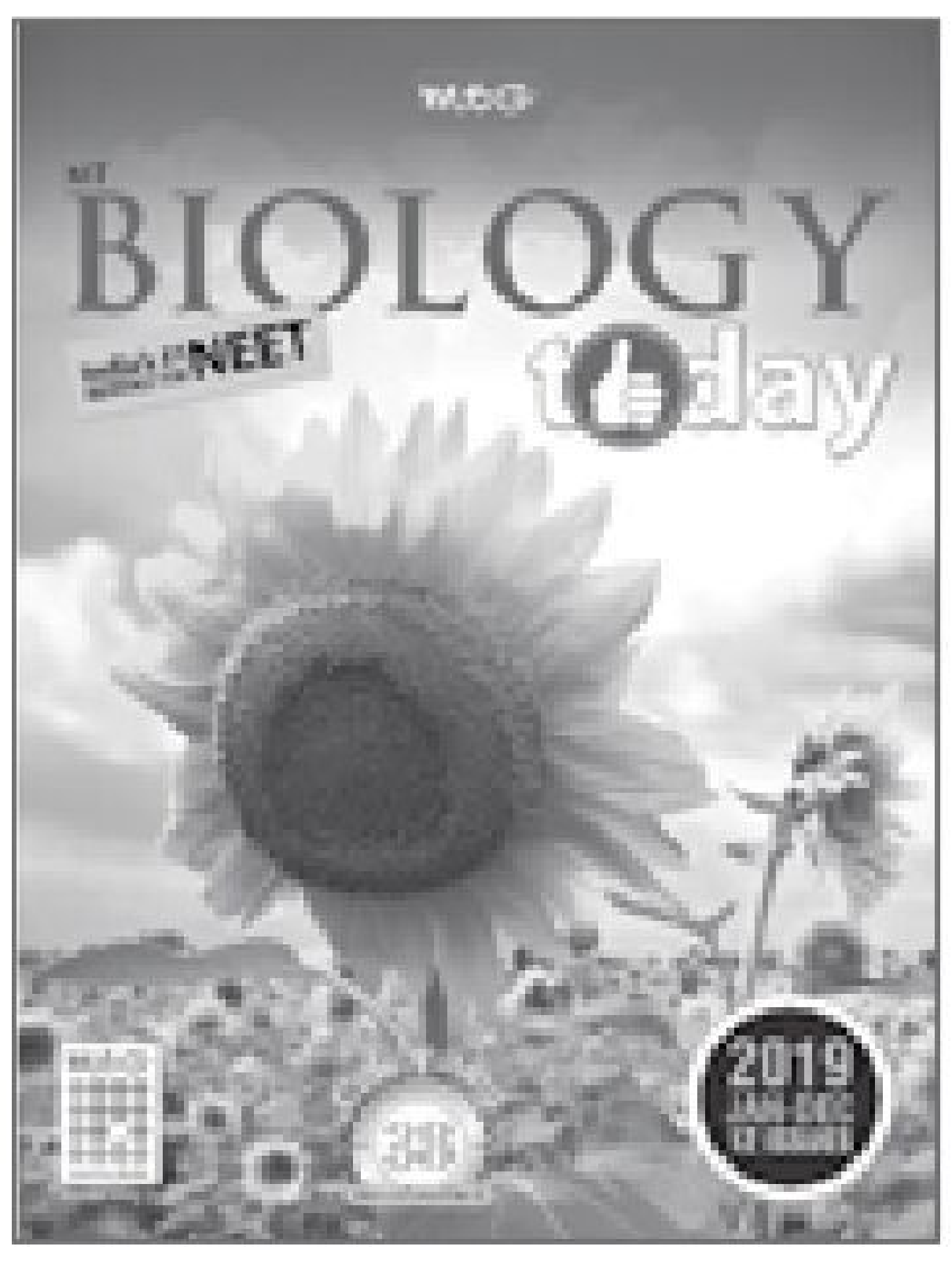

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Complex Numbers & Quadratic Equations

Single Correct Answer Type

1. If λ_1 and λ_2 be roots of the equation $x^2 + 2x + 2 = 0$ then least value of λ_3 for which $\left(\frac{\lambda_1}{\lambda_2}\right)^{\lambda_3} = 1$ is

(a) 2 (b) 3

(c) 4 (d) none of these

2. The product of the roots of the equation $|\sqrt{x}-3|+\sqrt{x}(\sqrt{x}-6)+7=0$ $(x > 0)$ is of the form 4^n then $n =$

(a) 4 (b) 3 (c) 2 (d) 1

3. The sum of all real values of x satisfying the equation $3^{(x-2)(x^2+4x-45)} = 1$ is

(a) 10 (b) 45 (c) −18 (d) −2

4. If $\alpha, \beta \in C$ are distinct roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2013} + \beta^{2019}$ is

(a) 0 (b) 1 (c) < 0 (d) > 0

5. If $f(x)$ is a quadratic polynomial such that $f(1) + f(2) = 0$ and −2 is a root of $f(x) = 0$, then other root of $f(x) = 0$ is

(a) 11/7 (b) −11/7 (c) 7/11 (d) −7/11

6. The sum of the roots of the equation $4^{36x-2} + 4^{12x+2} = 4^{24x+1} + 1$ is

(a) 1/12 (b) 1/6 (c) 1/4 (d) 1/2

7. Let $z(\alpha, \beta) = \cos\alpha + e^{i\beta}\sin\alpha$, $(\alpha, \beta \in R,\ i = \sqrt{-1})$ then the exhaustive set of values of modulus of $z(\theta, 2\theta)$ as θ varies, is

(a) $\left[\frac{1}{2}, \frac{3}{2}\right]$ (b) $[-\sqrt{2}, \sqrt{2}]$

(c) $\left[-\sqrt{\frac{3}{2}}, \frac{-1}{\sqrt{2}}\right]$ (d) $\left[\frac{1}{\sqrt{2}}, \sqrt{\frac{3}{2}}\right]$

8. The sum of infinite terms of the series $\sin\frac{\pi}{3}+\frac{1}{2}\sin\frac{2\pi}{3}+\frac{1}{3}\sin\frac{3\pi}{3}+\frac{1}{4}\sin\frac{4\pi}{3}+\ \ldots.\infty$ equals

(a) π/12 (b) π/3 (c) π/15 (d) 0

9. If the complex number z for which $\arg\left(\frac{3z-9-3i}{4z-20+8i}\right)=\frac{\pi}{4}$ and $|z - 1 + 2i| = 2$ are $\left(a-\frac{2}{\sqrt{10}}\right)+i\left(b+\frac{6}{\sqrt{10}}\right)$ and $\left(a+\frac{2}{\sqrt{10}}\right)+i\left(b-\frac{6}{\sqrt{10}}\right)$ respectively then set of ordered pair (a, b) is

(a) (1, 2) (b) (−1, −2)

(c) (1, −2) (d) (−2, 1)

10. Let α, β are roots of the equation $x^2 - 8ax + 3 - 3a + 16a^2 = 0$. The value of '$a$' for which roots exceed 4 is

(a) $>\frac{19}{16}$ (b) $<\frac{19}{16}$

(c) <1 (d) none of these

More Than One Correct Answer Type

11. If two roots of the equation $(a - 2)(x^4 + x^2 + 1) + (a + 2)(x^2 + x + 1)^2 = 0$ are real and unequal, then set of all values of 'a 'is

(a) $(-1, 0)$ (b) $(-1, 0) \cup (0, 1)$
(c) $(-1, 1) - \{0\}$ (d) $(0, 1)$

12. The inequality $\left|\dfrac{x^2+mx+1}{x^2+x+1}\right|<3$ is satisfied for all real values of x, then
(a) $-1 < m < 5$
(b) Number of integral values of m except 0 is four.
(c) The equation $x^2 - (m^2 - 4m - 5)\,x - 1 = 0$ has exactly one real root in (0, 1).
(d) $-5 < m < 1$

13. The roots of the equation,
$(x^2 + 1)^2 = x(4x^2 + 5x + 4)$ are both real and imaginary of the form $A\pm\sqrt{B}$ and $C\pm iD$ respectively, then
(a) $D=\dfrac{\sqrt{3}}{2}$ (b) $A=\dfrac{5}{2}$
(c) $B=\dfrac{21}{4}$ (d) $C=-\dfrac{1}{2}$

14. If $a, b, c \in R$ such that their sum is zero, then the quadratic equation $3ax^2 + 2bx + c = 0$ has
(a) at least one root in $[-2, 1]$
(b) at least one root in $[0, 2]$
(c) at least one root in $[0, 1]$
(d) at least one root in $[-\infty, \infty]$

15. Let $f(x) = Ax^2 + Bx + C$, $A, B, C \in R$ and $A \neq 0$, let $f(x)$ is positive for all real values.
If $g(x) = f(x) + f'(x) + f''(x)$, then
(a) $g(x)$ is positive $\forall\ x \in R$
(b) $g(x)$ is negative $\forall\ x \in R$
(c) $g(x) = 0$ has non real complex roots
(d) $g(x) = 0$ has real roots

16. It is given that the complex numbers which satisfy the equation $z\bar{z}^3+\bar{z}z^3=350$ form a rectangle in the Guassian plane with the length of its diagonal having an integral number of units then
(a) length of diagonals is 10 units.
(b) area of rectangle is 48 sq. units.
(c) rectangle is symmetrical about both axes.
(d) if z_1, z_2, z_3, z_4 are vertices of a rectangle, then $\sum_{i=1}^{4} z_i = 0$.

17. Let $x_1 = z_1 + z_2 + z_3$, $x_2 = z_1 + \omega z_2 + \omega^2 z_3$ and $x_3 = z_1 + \omega^2 z_2 + \omega z_3$, where $1, \omega, \omega^2$ are cube roots of unity, then
(a) $z_1=\dfrac{x_1+x_2+x_3}{3}$
(b) $z_2=\dfrac{x_1+\omega^2 x_2+\omega x_3}{3}$
(c) $z_3=\dfrac{x_1+\omega x_2+\omega^2 x_3}{3}$
(d) $z_1 z_2 z_3 = x_1^2 + x_2^2 + x_3^2 - (x_1x_2 + x_2x_3 + x_1x_3)$

Comprehension Type

Paragraph for Q. No. 18 to 20

Consider quadratic equation $ax^2 + bx + c = 0$ ....(1) where $a \neq 0$, $a, b, c, \in R$. If α, β are roots of equation of (1) then sum of the roots equal to negative of the coefficient of x divided by coefficient of x^2 and product of roots of (1) is the ratio of constant term to the coefficient of x^2. The equation (1) can be rewrite as $ax^2 + bx + c = a(x - \alpha)(x - \beta)$. Again, if $a_i(i = 1, 2, 3, ..., n)$ are in A.P. then difference between the consecutive terms is same which may be zero or a non zero quantity, if b_i $(i = 1, 2, 3, ..., n)$ are in G.P. then $\dfrac{b_2}{b_1}=\dfrac{b_3}{b_2}=.....=\dfrac{b_n}{b_{n-1}}$ is a fixed number may be equal to 1, less than 1 or greater than 1 and if c_i $(i = 1, 2, ..., n)$ are in H.P. then $\dfrac{1}{c_2}-\dfrac{1}{c_1}=\dfrac{1}{c_3}-\dfrac{1}{c_2}=....=\dfrac{1}{c_n}-\dfrac{1}{c_{n-1}}$ is also a fixed quantity.On the basis of above information, answer the following questions.

18. Let p and q be the roots of the equation $x^2 - 4x + \alpha_1 = 0$ and r and s be the roots of the equation $x^2 - 36x + \alpha_2 = 0$ such that $p < q < r < s$ are in A.P., then values of α_1 and α_2 respectively are
(a) $-12, 308$ (b) $12, 308$
(c) $12, -6$ (d) $12, -308$

19. Let l_1, l_2 be the roots of equation $x^2 - 2x + A = 0$ and l_3, l_4 be the roots of equation $x^2 - 8x + B = 0$ such that l_1, l_2, l_3, l_4 are in G.P., then integral values of A and B respectively are
(a) $8, 128$ (b) $-8, -128$
(c) $-8, 128$ (d) $8, -128$

20. If β_1, β_3 are roots of equation $Ax^2 - 6x + 1 = 0$ and β_2, β_4 are roots of $Bx^2 - 10x + 1 = 0$ such that $\beta_1, \beta_2, \beta_3, \beta_4$ are in H.P., then values of A and B respectively are
(a) $5, -21$ (b) $-5, -21$
(c) $5, 21$ (d) $-5, 21$

Paragraph for Q. No. 21 to 23

$\sqrt{-1}=i$ called imaginary unit such that sum of consecutive four powers (*i.e.*, $i^n + i^{n+1} + i^{n+2} + i^{n+3}$) is zero $\forall\ n \in I$. Also $x = (1)^{1/3} \Rightarrow x^3 - 1 = 0$
$\Rightarrow (x - 1)(x^2 + x + 1) = 0$
$\Rightarrow (x - 1)(x - \omega)(x - \omega^2) = 0$, *i.e.*, $x = 1, \omega, \omega^2$ are cube roots of unity and imaginary roots of $x = (1)^{1/3}$ satisfies $\omega^n + \omega^{n+1} + \omega^{n+2} = 0\ \forall\ n \in I$.

Also $1\cdot\omega\cdot\omega^2 = \omega^3 = 1$.

21. If $(1+\omega)(1+\omega^2)(1+\omega^3)(1+\omega^4)\ \ldots.\ (1+\omega^{6057}) = 2^{\lambda}$ then the value of λ equals

(a) 2019 (b) 2018

(c) 2017 (d) 2016

22. If $\omega(\neq 1)$ is a cube root of unity then the value of $\frac{1}{\sqrt{3}}\,|1+2\omega+3\omega^2+\ldots.+(3n-1)\omega^{3n-2}+3n\omega^{3n-1}|$ $\forall\, n \in N$, can not exceed by

(a) $n+2$ (b) $n+1$

(c) n (d) 2

23. If $z = \frac{\sqrt{3}+i}{2}$, then value of $(z^{113}+i^{163})^{235} =$

(a) z (b) z^2

(c) z^3 (d) z^4

Matrix Match Type

24. Match the following :

	Column-I		Column-II
A.	If n^{th} roots of unity are $1, \omega, \omega^2, \omega^3, \ldots, \omega^{n-1}$, then $(2-\omega)(2-\omega^2)\ \ldots.\ (2-\omega^{n-1})$ equals	P.	${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \ldots$
B.	If z_i $(i = 1, 2, \ldots, n)$ lies on $\lvert z\rvert = 2$, then value of $\left\lvert\sum_{i=1}^{n} z_i\right\rvert - 4\left\lvert\sum_{i=1}^{n}\frac{1}{z_i}\right\rvert$ equals	Q.	$2^n - 1$
C.	If $\alpha_0, \alpha_1, \alpha_2, \ldots., \alpha_{n-1}$ be the n, n^{th} roots of unity, then the value of $\sum_{i=0}^{n-1}\frac{\alpha_i}{3-\alpha_i}$ is equal to	R.	${}^nC_{n-1} + {}^nC_{n-2} + \ldots. + {}^nC_0$
		S.	$\frac{n}{3^n-1}$
		T.	$\sqrt{\left({}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots + {}^{2n+1}C_n\right)} - 1$
		U.	$\sqrt{{}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + \ldots + {}^{2n+1}C_{2n+1}} - 1$

Numerical Answer Type

25. Let S be the greatest integral value of the sum of roots of equation $x^{2019} + \left(\frac{1}{3} - x\right)^{2019} = 0$ taken two at a time then sum of the digits of S equals __________.

26. Find the sum of all divisors of the sum of all integral values of α for which one root of the equation $(\alpha - 5)x^2 - 2\alpha x + \alpha - 4 = 0$ is smaller than 1 and other is greater than 2.

27. Let $f(x) = x^2 - ax + b$, a is an odd positive integer and the roots of the equation $f(x) = 0$ are two non-equal prime numbers. If it is known that $a + b = 17$, then $f(4)\sum_{r=1}^{4} f(r)$ must be equal to __________.

28. If $x = a + b\omega + c\omega^2$, $y = a\omega + b\omega^2 + c$ and $z = a\omega^2 + b + c\omega$, then the value of $\frac{x^2}{yz} + \frac{y^2}{xz} + \frac{z^2}{xy}$ is equal to __________.

29. If $\sum_{a=1}^{20}(3a+4)\left[\sum_{b=1}^{6}\left\{\sin\left(\frac{2b\pi}{7}\right) - i\cos\left(\frac{2b\pi}{7}\right)\right\}\right]^{4a} = \alpha$ where $i = \sqrt{-1}$, then value of α equals __________.

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1. (d) **2.** (a) **3.** (c) **4.** (b) **5.** (b)

6. (d) **7.** (b,c) **8.** (a,b,c,d) **9.** (a,d)

10. (b,c,d) **11.** (a,b,c,d) **12.** (a,b) **13.** (c)

14. (d) **15.** (a) **16.** (b) **17.** (8) **18.** (4)

19. (2) **20.** (10)

30. If $z_m = \cos\left(\frac{m\pi}{10}\right) + i\sin\left(\frac{m\pi}{10}\right)$, then $\prod_{m=1}^{4} z_m$ is equal to ________.

SOLUTIONS

1. (c) : ∵ λ_1, λ_2 are roots of the equation $x^2 + 2x + 2 = 0$

$\Rightarrow (x + 1)^2 + 1 = 0 \Rightarrow (x + 1)^2 = -1 = i^2$

$\Rightarrow x + 1 = \pm i \Rightarrow x = -1 \pm i = -1 + i, -1 - i$

$\therefore \left(\frac{\lambda_1}{\lambda_2}\right)^{\lambda_3} = 1 \Rightarrow \left(\frac{-1+i}{-(1+i)}\right)^{\lambda_3} = 1$

[Considering $\lambda_1 = -1 + i$ and $\lambda_2 = -1 - i$]

$\Rightarrow \left(\frac{1-i}{1+i}\right)^{\lambda_3} = 1 \Rightarrow (-i)^{\lambda_3} = 1$

∴ Least value of λ_3 is 4.

2. (b) : We have $|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 7 = 0$

$\Rightarrow |\sqrt{x} - 3| + x - 6\sqrt{x} + 7 = 0$

$\Rightarrow |\sqrt{x} - 3| + x - 6\sqrt{x} + 9 + 7 - 9 = 0$

$\Rightarrow |\sqrt{x} - 3| + (\sqrt{x} - 3)^2 - 2 = 0$

$\Rightarrow |\sqrt{x} - 3|^2 + |\sqrt{x} - 3| - 2 = 0$

$\Rightarrow \lambda^2 + \lambda - 2 = 0$, where $\lambda = |\sqrt{x} - 3|$

$\Rightarrow (\lambda + 2)(\lambda - 1) = 0 \Rightarrow \lambda = 1, \lambda = -2$

$\Rightarrow |\sqrt{x} - 3| = 1$, $|\sqrt{x} - 3| = -2$, which is not possible

$\Rightarrow \sqrt{x} - 3 = \pm 1 \Rightarrow \sqrt{x} = 4$ and $\sqrt{x} = 2$

$\Rightarrow x = 16, x = 4$

∴ Product of roots $= 16 \times 4 = 4^3 = 4^n$

$\therefore n = 3$

3. (d) : $3^{(x-2)(x^2+4x-45)} = 1 = 3^0$

$\Rightarrow (x - 2)(x^2 + 4x - 45) = 0$

$\Rightarrow (x - 2)(x + 9)(x - 5) = 0 \Rightarrow x = 2, 5, -9 \in R$

∴ Required sum of all real values $= 2 + 5 - 9 = -2$

4. (c) : ∵ $x^2 - x + 1 = 0$

$\Rightarrow x^2 + (\omega + \omega^2)x + 1 = 0$ (using $\omega + \omega^2 = -1$)

$\Rightarrow x(x + \omega) + \omega^2(x + \omega) = 0$

$\Rightarrow (x + \omega)(x + \omega^2) = 0 \Rightarrow x = -\omega, -\omega^2$

$\therefore (-\omega)^{2013} + (-\omega^2)^{2019} = -[1 + 1] = -2 < 0$

or $(-\omega^2)^{2013} + (-\omega)^{2019} = -[1 + 1] = -2 < 0$

$\therefore \alpha^{2013} + \beta^{2019} = \alpha^{2019} + \beta^{2013} = -2 < 0$

5. (a) : Let $f(x) = ax^2 + bx + c$ ∴ $f(1) = a + b + c$, $f(2) = 4a + 2b + c$

∴ $f(1) + f(2) = 0 \Rightarrow 5a + 3b + 2c = 0$ ...(i)

Since -2 is a root of $f(x) = 0$.

∴ $a(-2)^2 + b(-2) + c = 0 \Rightarrow 4a - 2b + c = 0$ ...(ii)

From (i) and (ii), we get

$3a - 7b = 0 \Rightarrow \frac{b}{a} = \frac{3}{7}$

If other root of $f(x) = 0$ is α, then sum of roots

$= \alpha + (-2) = -\frac{b}{a} \Rightarrow \alpha - 2 = \frac{-3}{7} \Rightarrow \alpha = \frac{-3}{7} + 2 = \frac{11}{7}$

6. (b) : Let $4^{12x} = t$

$\therefore 4^{36x-2} + 4^{12x+2} = 4^{24x+1} + 1$

$\Rightarrow \frac{t^3}{16} + 16t = 4t^2 + 1$

$\Rightarrow t^3 - 64t^2 + 256t - 16 = 0$ ...(i)

Let (i) has roots t_1, t_2, t_3, then

$t_1 t_2 t_3 = \frac{-\text{constant term}}{\text{coefficient of } t^3} = 16$

$\Rightarrow 4^{12x_1} \cdot 4^{12x_2} \cdot 4^{12x_3} = 4^2$

$\Rightarrow 4^{12(x_1+x_2+x_3)} = 4^2 \Rightarrow x_1 + x_2 + x_3 = \frac{1}{6}$

∴ Sum of roots $= \frac{1}{6}$

7. (d) : ∵ $z(\alpha, \beta) = \cos\alpha + e^{i\beta}\sin\alpha$

$\therefore z(\theta, 2\theta) = \cos\theta + e^{i2\theta}\sin\theta = \cos\theta + (\cos 2\theta + i\sin 2\theta)\sin\theta$
$= (\cos\theta + \cos 2\theta \sin\theta) + i\sin 2\theta \sin\theta$

$\Rightarrow |z(\theta, 2\theta)| = \sqrt{(\cos\theta + \cos 2\theta \sin\theta)^2 + (\sin 2\theta \sin\theta)^2}$

$= \sqrt{\cos^2\theta + \sin^2\theta(\cos^2 2\theta + \sin^2 2\theta) + \frac{\sin 4\theta}{2}} = \sqrt{1 + \frac{\sin 4\theta}{2}}$

Now, $-1 \le \sin 4\theta \le 1 \Rightarrow -\frac{1}{2} \le \frac{\sin 4\theta}{2} \le \frac{1}{2}$

$\Rightarrow \frac{1}{2} \le 1 + \frac{\sin 4\theta}{2} \le \frac{3}{2} \Rightarrow \frac{1}{\sqrt{2}} \le \sqrt{1 + \frac{\sin 4\theta}{2}} \le \sqrt{\frac{3}{2}}$

$\Rightarrow |z(\theta, 2\theta)| \in \left[\frac{1}{\sqrt{2}}, \sqrt{\frac{3}{2}}\right]$

8. (b) : Let $S = \sin\frac{\pi}{3} + \frac{1}{2}\sin\frac{2\pi}{3} + \frac{1}{3}\sin\frac{3\pi}{3} + \ldots\ldots \infty$

and $C = \cos\frac{\pi}{3} + \frac{1}{2}\cos\frac{2\pi}{3} + \frac{1}{3}\cos\frac{3\pi}{3} + \ldots\ldots \infty$

$$\therefore C + iS = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) + \frac{1}{2}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) + \frac{1}{3}\left(\cos\frac{3\pi}{3} + i\sin\frac{3\pi}{3}\right) + \ldots\infty$$

$$= e^{i\frac{\pi}{3}} + \frac{1}{2}e^{i\frac{2\pi}{3}} + \frac{1}{3}e^{i\frac{3\pi}{3}} + \ldots\infty$$

$$= y + \frac{y^2}{2} + \frac{y^3}{3} + \dots\infty \quad \left(\text{where } y = e^{i\frac{\pi}{3}}\right)$$

$= -\log(1 - y)$

$$= -\log\left(1 - e^{i\frac{\pi}{3}}\right) = -\log\left(1 - \cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)$$

$$= -\log\left[\left(1 - \frac{1}{2}\right) - i\frac{\sqrt{3}}{2}\right] \quad = -\log\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$$

$$= -\log z \quad \left(\text{where } z = \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)$$

$= -[\log|z| + i\arg(z)]$ (using $\log z = \log|z| + i\arg(z)$)

$$= -\left[\log 1 + i\tan^{-1}\left(\frac{\operatorname{Im} z}{\operatorname{Re} z}\right)\right] = 0 - i\tan^{-1}\left(\frac{-\sqrt{3}/2}{1/2}\right)$$

$$= -i\tan^{-1}(-\sqrt{3}) = i\tan^{-1}(\sqrt{3}) = i\frac{\pi}{3}$$

$$\therefore\ C + iS = i\frac{\pi}{3} \Rightarrow S = \frac{\pi}{3}$$

9. (c) : Given, $\arg\left(\dfrac{3z - 9 - 3i}{4z - 20 + 8i}\right) = \dfrac{\pi}{4}$

$$\Rightarrow \arg\left[\frac{3}{4}\left(\frac{z - 3 - i}{z - 5 + 2i}\right)\right] = \frac{\pi}{4}$$

$$\Rightarrow \arg\left(\frac{3}{4}\right) + \arg\left(\frac{z - 3 - i}{z - 5 + 2i}\right) = \frac{\pi}{4}$$

$$\Rightarrow \arg\left(\frac{z - 3 - i}{z - 5 + 2i}\right) = \frac{\pi}{4}$$

($\because$ Argument of a real number = 0)

$$\Rightarrow \arg\left[\frac{(x-3) + i(y-1)}{(x-5) + i(y+2)}\right] = \frac{\pi}{4} \qquad (\text{where } z = x + iy)$$

$$\Rightarrow \arg\left[\frac{[(x-3) + i(y-1)][(x-5) - i(y+2)]}{(x-5)^2 + (y+2)^2}\right] = \frac{\pi}{4}$$

$$\Rightarrow \arg\left[\frac{[(x-3)(x-5) + (y-1)(y+2)] + i[(y-1)(x-5) - (x-3)(y+2)]}{(x-5)^2 + (y+2)^2}\right] = \frac{\pi}{4}$$

$\Rightarrow\ (y - 1)(x - 5) - (x - 3)(y + 2) = (x - 3)(x - 5) + (y - 1)(y + 2)$

$\Rightarrow\ x^2 + y^2 - 5x + 3y + 2 = 0$ ...(1)

and $|z - 1 + 2i| = 2 \Rightarrow (x - 1)^2 + (y + 2)^2 = 4$

$\Rightarrow\ x^2 + y^2 - 2x + 4y + 1 = 0$ ...(2)

Now, from (1) and (2) on subtracting or equating, we get $3x + y - 1 = 0 \Rightarrow y = 1 - 3x$ ...(3)

Now, from (1) and (3), we get

$x^2 + (1 - 3x)^2 - 5x + 3(1 - 3x) + 2 = 0$

$\Rightarrow\ 10x^2 - 20x + 6 = 0 \Rightarrow 5x^2 - 10x + 3 = 0$

$$\Rightarrow\ x = \frac{10 \pm 2\sqrt{10}}{10}$$

Now, using $y = 1 - 3x$, we get

$$y = \frac{-20 \mp 6\sqrt{10}}{10} = -2 \mp \frac{6}{\sqrt{10}}$$

$$\Rightarrow\ x = 1 \pm \frac{2}{\sqrt{10}} \text{ and } y = -2 \mp \frac{6}{\sqrt{10}}$$

$$\therefore\ x + iy = \left(1 \pm \frac{2}{\sqrt{10}}\right) + i\left(-2 \mp \frac{6}{\sqrt{10}}\right)$$

$$= \left(a \pm \frac{2}{\sqrt{10}}\right) + i\left(b \mp \frac{6}{\sqrt{10}}\right) \Rightarrow a = 1,\ b = -2$$

10. (a) : Using concept, the function $f(x) = Ax^2 + Bx + C = 0$ has both roots greater than the real value k if

(i) $D > 0$ (ii) $\dfrac{\alpha + \beta}{2} > k$ (iii) $A\,f(k) > 0$

Now (i) $D > 0 \Rightarrow 64a^2 - 4(1)(3 - 3a + 16a^2) > 0$

$\Rightarrow\ a > 1$

(ii) $\dfrac{\alpha + \beta}{2} > 4 \Rightarrow \dfrac{8a}{2} > 4 \Rightarrow a > 1$

(iii) $Af(k) > 0 \Rightarrow 1 \cdot f(4) > 0$

$\Rightarrow\ 1(16 - 32a + 3 - 3a + 16a^2) > 0$

$\Rightarrow\ 16a^2 - 35a + 19 > 0 \Rightarrow (16a - 19)(a - 1) > 0$

$\Rightarrow\ a < 1$ or $a > \dfrac{19}{16}$ $\quad\therefore$ (i), (ii) and (iii) $\Rightarrow a > \dfrac{19}{16}$

11. (b, c) : We have, $(a - 2)(x^4 + x^2 + 1) + (a + 2)(x^2 + x + 1)^2 = 0$

$\Rightarrow\ (a - 2)(x^2 + x + 1)(x^2 - x + 1) + (a + 2)(x^2 + x + 1)^2 = 0$

$\Rightarrow\ (x^2 + x + 1)[(a - 2)(x^2 - x + 1) + (a + 2)(x^2 + x + 1)] = 0$

$\Rightarrow\ (x^2 + x + 1) = 0$ or $(a - 2)(x^2 - x + 1) + (a + 2)(x^2 + x + 1) = 0$

$\Rightarrow\ x^2 + x + 1 = 0$ or $a(x^2 - x + 1 + x^2 + x + 1) - 2(x^2 - x + 1 - x^2 - x - 1) = 0$

$\Rightarrow\ 2a(x^2 + 1) + 2(2x) = 0$

$\Rightarrow\ ax^2 + 2x + a = 0\ (a \neq 0)$ ....(i)

For real and unequal roots of (i), we have

$4(1 - a^2) > 0 \Rightarrow |a| < 1 \Rightarrow -1 < a < 1$ ....(ii)

$\therefore\ a \in (-1, 1) - \{0\}$ or $(-1, 0) \cup (0, 1)$

12. (a, b, c) : From given inequality, we have

$$-3 < \frac{x^2 + mx + 1}{x^2 + x + 1} < 3$$

$\because\ x^2+x+1=\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$ is $\geq\frac{3}{4}$

$\Rightarrow\ -3(x^2+x+1) < x^2+mx+1 < 3(x^2+x+1)$

$\Rightarrow\ 4x^2+(m+3)x+4>0$ ...(1)

and $2x^2+(3-m)x+2>0\ \forall\ x\in R$ ...(2)

In both cases $f(x)>0$ and $a>0$ therefore discriminant must be negative, where a is coefficient of x^2.

$\therefore\ D_1<0$ and $D_2<0$

$\Rightarrow\ (m+3)^2-64<0$ and $(3-m)^2-16<0$

$\Rightarrow\ m^2+6m-55<0$ and $m^2-6m-7<0$

$\Rightarrow\ (m+11)(m-5)<0$ and $(m-7)(m+1)<0$

$\Rightarrow\ m\in(-11,5)$ and $m\in(-1,7)$

$\Rightarrow\ m\in(-1,5)$ (option (a) is correct)

Now, integral values of $m = 0, 1, 2, 3, 4$

(option (b) is correct)

Further let $f(x)=x^2-(m^2-4m-5)x-1$

Then, $f(0)=-1$ and $f(1)=-(m^2-4m-5)$

$\therefore\ f(0)\,f(1)=(-1)^2(m^2-4m-5)$

$=(m^2-4m-5)=(m-5)(m+1)<0\ \forall\ m\in(-1,5)$

$\therefore\ f(x)=0$ has only one real root in $(0, 1)$.

(option (c) is correct)

13. (a, b, c, d) : Given equation is

$(x^2+1)^2=x(4x^2+5x+4)$ ...(i)

$\Rightarrow\ x^4-4x^3-3x^2-4x+1=0$

$$\Rightarrow\ x^2\left(x^2-4x-3-\frac{4}{x}+\frac{1}{x^2}\right)=0$$

$$\Rightarrow\ \left(x^2+\frac{1}{x^2}\right)-4\left(x+\frac{1}{x}\right)-3=0$$

(as $x=0$ does not satisfy (i))

$$\Rightarrow\ t^2-4t-5=0,\text{ where } t=x+\frac{1}{x}$$

$$\Rightarrow\ (t+1)(t-5)=0\ \Rightarrow\ \left(x+\frac{1}{x}+1\right)\left(x+\frac{1}{x}-5\right)=0$$

$\Rightarrow\ (x^2+x+1)(x^2-5x+1)=0$

$\Rightarrow\ x^2+x+1=0,\ x^2-5x+1=0$

$$\Rightarrow\ x=\frac{-1\pm i\sqrt{3}}{2},\ x=\frac{5\pm\sqrt{21}}{2}$$

$$x=-\frac{1}{2}\pm i\frac{\sqrt{3}}{2}=C\pm iD,\ x=\frac{5}{2}\pm\sqrt{\frac{21}{4}}=A\pm\sqrt{B}$$

$$\Rightarrow C=\frac{-1}{2}, D=\frac{\sqrt{3}}{2},\quad A=\frac{5}{2},\ B=\frac{21}{4}$$

$\therefore$ All options are correct.

14. (a, b, c, d) : Let $f(x)=ax^3+bx^2+cx+d=0$, $f(x)$ is a polynomial function and so f is continous and differentiable on R.

Also, $f(0)=d$ and $f(1)=a+b+c+d=d$

$(\because\ a+b+c=0)$

$\Rightarrow\ f(0)=f(1)\Rightarrow$ Rolle's theorem is applicable.

Thus there exist at least one value of $\alpha\in(0,1)$ for which $f'(\alpha)=0$

Here, $f'(\alpha)=0\Rightarrow 3a\alpha^2+2b\alpha+c=0$

$\Rightarrow\ 3ax^2+2bx+c=0$ has at least one root in $[0, 1]$

Again, $[0,1]\subseteq[-2,1]$, $[0,1]\subseteq[0,2]$, $[0,1]\subseteq(-\infty,\infty)$

$\therefore$ All options are correct.

15. (a, c) : $f(x)$ is positive for all $x\in R$

$\therefore\ Ax^2+Bx+C>0$

$\Rightarrow\ B^2-4AC<0$ and $A>0$ ...(i)

Now, $f'(x)=2Ax+B,\ f''(x)=2A$

$\therefore\ g(x)=Ax^2+Bx+C+2Ax+B+2A$

$=Ax^2+(2A+B)x+(2A+B+C)$

Here, we have $A>0$ and $D=(2A+B)^2-4A(2A+B+C)=(B^2-4AC)-4A^2$ = sum of nagative numbers <0 (using (i))

$\therefore$ For $g(x)$ we have, coefficient of $x^2>0$ and discriminant <0

$\Rightarrow\ g(x)>0\ \forall\ x\in R\Rightarrow g(x)=0$ has non real complex roots.

16. (a, b, c, d) : Let the complex number be $z=x+iy$, which satisfy the given equation $z\bar{z}^3+\bar{z}z^3=350$

$$\Rightarrow\ z\,\bar{z}\,\bar{z}^2+\bar{z}\,z\,z^2=350$$

$$\Rightarrow\ z\,\bar{z}(\bar{z}^2+z^2)=350$$

$\Rightarrow\ (x^2+y^2)\,[(x-iy)^2+(x+iy)^2]=350$

$\Rightarrow\ 2(x^2+y^2)\,(x^2-y^2)=350$

$\Rightarrow\ (x^2+y^2)\,(x^2-y^2)=175=25\times 7=5\times 35$

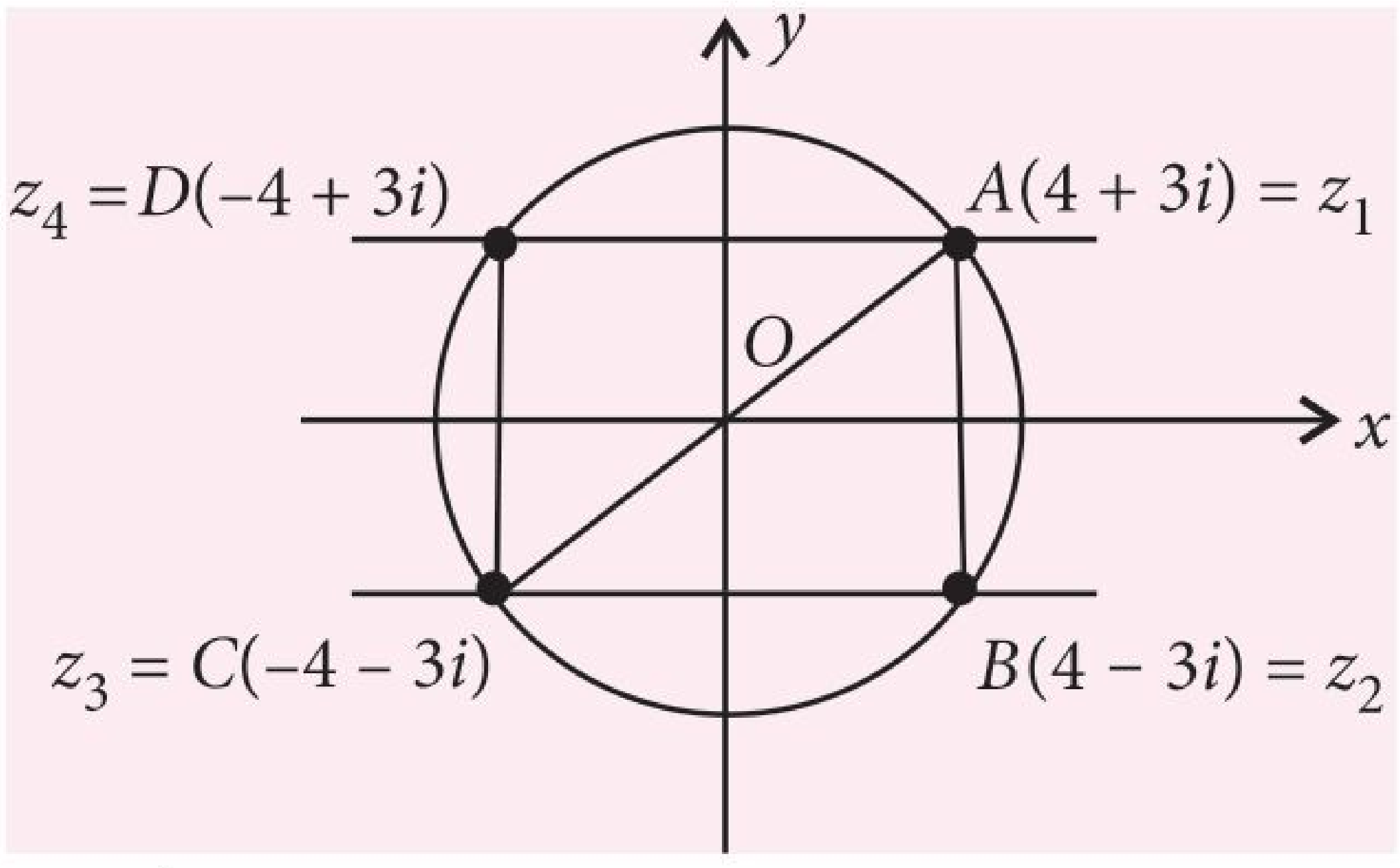
$\Rightarrow\ x^2+y^2=25$ and $x^2-y^2=7$

$\Rightarrow\ x^2=16,\ y^2=9\ \ \therefore\ \ x=\pm 4,\ y=\pm 3$

[The other possibilities does not hold the conditions]

$\therefore\ x+iy=4+3i,\ 4-3i,\ -4-3i,\ -4+3i,$

$\therefore$ Length of diagonal $=\sqrt{64+36}=10$ units



$$\text{Consider, }\sum_{i=1}^{4}z_i=(4+3i)+(4-3i)+(-4-3i)+(-4+3i)=0$$

Area of rectangle $=6\times 8=48$ sq. units

From the figure, rectangle is symmetrical about both axes.

$\therefore$ All options are correct

17. (a, b, c) : We have, $x_1 = z_1 + z_2 + z_3$ ....(i)

$x_2 = z_1 + \omega z_2 + \omega^2 z_3$ ...(ii)

and $x_3 = z_1 + \omega^2 z_2 + \omega z_3$ ...(iii)

On adding (i), (ii) and (iii), we get

$x_1 + x_2 + x_3 = 3z_1 + z_2(1 + \omega + \omega^2) + z_3(1 + \omega + \omega^2)$

$\Rightarrow x_1 + x_2 + x_3 = 3z_1$ $(\because 1 + \omega + \omega^2 = 0)$

$$\Rightarrow z_1 = \frac{x_1 + x_2 + x_3}{3}$$

Again, multiplying (i), (ii) and (iii) by 1, ω^2, ω respectively and then adding, we get

$z_1(1 + \omega^2 + \omega) + z_2(1 + \omega^3 + \omega^3) + z_3(1 + \omega^4 + \omega^2)$

$= x_1 + \omega^2 x_2 + \omega x_3$

$$\Rightarrow z_2 = \frac{x_1 + \omega^2 x_2 + \omega x_3}{3}$$

Similarly, $z_3 = \dfrac{x_1 + \omega x_2 + \omega^2 x_3}{3}$

and $z_2 z_3 = \dfrac{1}{9}(x_1^2 + x_2^2 + x_3^2 - x_1x_2 - x_2x_3 - x_3x_1)$

18. (a) : As $p < q < r < s \in$ A.P.

$\therefore$ $q = p + D$, $r = p + 2D$, $s = p + 3D$ where D is common difference.

Now, as p, q are roots of $x^2 - 4x + \alpha_1 = 0$

$\therefore$ $p + q = 4$ and $pq = \alpha_1 \Rightarrow 2p + D = 4$ ...(i)

and as r, s are roots of $x^2 - 36x + \alpha_2 = 0$

$\therefore$ $r + s = 36$ and $rs = \alpha_2$

$\Rightarrow 2p + 5D = 36$ ...(ii)

Solving (i) and (ii), we get $D = 8$, $p = -2$

$\therefore$ $q = p + D = 6$, $r = p + 2D = 14$, $s = p + 3D = 22$

Now, $\alpha_1 = pq = -12$ and $\alpha_2 = rs = 14 \times 22 = 308$

19. (b) : As $l_1, l_2, l_3, l_4 \in$ G.P.

$\therefore l_2 = l_1R$, $l_3 = l_1R^2$, $l^4 = l_1R^3$, where R is common ratio.

$\because$ l_1, l_2 are roots of $x^2 - 2x + A = 0$

$\Rightarrow l_1l_2 = A$ and $l_1 + l_2 = 2 \Rightarrow l_1(1 + R) = 2$ ...(1)

and $A = l_1l_2 = l_1^2R$ ...(*)

Again l_3, l_4 are roots of the equation $x^2 - 8x + B = 0$

$\therefore$ $l_3 + l_4 = l_1R^2(1 + R) = 8$ ...(2)

and $l_3l_4 = l_1^2R^5 = B$ ...(**)

On dividing (2) by (1), we get

$$\frac{l_1R^2(1+R)}{l_1(1+R)} = \frac{8}{2} \Rightarrow R^2 = 4 \Rightarrow R = \pm 2$$

Case (i) : If $R = 2$, then $l_1(1 + R) = 2 \Rightarrow l_1 = \dfrac{2}{3}$

$\therefore$ l_1, l_2, l_3, l_4 are $\dfrac{2}{3}, \dfrac{4}{3}, \dfrac{8}{3}, \dfrac{16}{3}$

$\Rightarrow$ $l_1l_2 = \dfrac{8}{9} = A$, $l_3l_4 = \dfrac{128}{9} = B$, rejecting the value of $R = 2$ as A and B do not satisfy the condition of integral values.

Case (ii) : If $R = -2$, then $l_1(1 + R) = 2 \Rightarrow l_1 = -2$

$\therefore$ l_1, l_2, l_3, l_4 are $-2, 4, -8, 16$

$\therefore$ $l_1l_2 = (-2)(4) = -8 = A$, which is an integer

and $l_3l_4 = (-8)(16) = -128 = B$, which is an integer

20. (c) : $\because$ β_1, β_3 are roots of $Ax^2 - 6x + 1 = 0$

$\therefore$ $\beta_1 + \beta_3 = \dfrac{6}{A}$ and $\beta_1\beta_3 = \dfrac{1}{A}$ ...(i)

$\Rightarrow$ $\dfrac{2\beta_1\beta_3}{\beta_1 + \beta_3} = \dfrac{1}{3}$ ...(*)

Similarly, $\beta_2 + \beta_4 = \dfrac{10}{B}$ and $\beta_2\beta_4 = \dfrac{1}{B}$ ...(ii)

$\Rightarrow$ $\dfrac{2\beta_2\beta_4}{\beta_2 + \beta_4} = \dfrac{1}{5}$ ...(**)

$\because$ $\beta_1, \beta_2, \beta_3, \beta_4 \in$ H.P.

$\Rightarrow$ $\beta_2 = \dfrac{2\beta_1\beta_3}{\beta_1 + \beta_3} = \dfrac{1}{3}$ which is a root of $Bx^2 - 10x + 1 = 0$

$$\therefore \quad B\left(\frac{1}{3}\right)^2 - 10\left(\frac{1}{3}\right) + 1 = 0 \Rightarrow B = 21$$

and $\beta_3 = \dfrac{2\beta_2\beta_4}{\beta_2 + \beta_4} = \dfrac{1}{5}$, which is a root of $Ax^2 - 6x + 1 = 0$

$$\Rightarrow \quad A\left(\frac{1}{5}\right)^2 - 6\left(\frac{1}{5}\right) + 1 = 0 \Rightarrow A = 5$$

21. (a) : Given, $(1 + \omega^1)(1 + \omega^2)(1 + \omega^3)(1 + \omega^4)$ .... $(1 + \omega^{6056})(1 + \omega^{6057}) = 2^\lambda$

Taking groups of three numbers we get 2019 groups.

$$\therefore \quad \underbrace{(1+\omega)(1+\omega^2)(1+\omega^3)}_{1^{\text{st}}\text{ group}}\underbrace{(1+\omega^4)(1+\omega^5)(1+\omega^6)}_{2^{\text{nd}}\text{ group}}$$

$$\ldots.\underbrace{(1+\omega^{6055})(1+\omega^{6056})(1+\omega^{6057})}_{(2019)^{\text{th}}\text{ group}}$$

$$= \underbrace{(-\omega^2)(-\omega)(2)}\underbrace{(-\omega^2)(-\omega)(2)}\ldots \text{ up to 2019 groups}$$

$= 2 \times 2 \times 2 \times$ .... (2019 times) $= 2^{2019} = 2^\lambda \Rightarrow \lambda = 2019$

22. (c) : Let $S = 1 + 2\omega + 3\omega^2 + \ldots. + (3n - 1)\omega^{3n-2} + 3n\omega^{3n-1}$ ...(i)

$\Rightarrow$ $\omega S = \omega + 2\omega^2 + \ldots. + (3n - 2)\omega^{3n-2} + (3n - 1)\omega^{3n-1} + 3n\omega^{3n}$ ...(ii)

Subtracting (ii) from (i), we get

$S(1 - \omega) = 1 + \omega + \omega^2 + \omega^3 + \ldots. + \omega^{3n-1} - 3n\omega^{3n}$

$= \dfrac{\omega^{3n} - 1}{\omega - 1} - 3n\omega^{3n}$ (using sum of G.P.)

$$\therefore \quad S = \frac{-3n}{1 - \omega} \Rightarrow |S| = \frac{3n}{|1 - \omega|} \Rightarrow |S| = \frac{3n}{\sqrt{3}} = \sqrt{3}n$$

$$\left\{\because\ |1-\omega|=\left|1-\left(\frac{-1}{2}\pm i\frac{\sqrt{3}}{2}\right)\right|=\left|\frac{3}{2}\mp\frac{i\sqrt{3}}{2}\right|=\sqrt{3}\right\}$$

$\Rightarrow\quad \dfrac{|S|}{\sqrt{3}}=n$ *i.e.*, can not be greater than n.

23. (a) : Given, $z=\dfrac{i+\sqrt{3}}{2}=\dfrac{i(i+\sqrt{3})}{2i}=\dfrac{1}{i}\left(\dfrac{i^2+i\sqrt{3}}{2}\right)$

$$=\frac{1}{i}\left(\frac{-1}{2}+i\frac{\sqrt{3}}{2}\right)$$

$\therefore\quad z=\dfrac{\omega}{i}$ or $z=-i\omega$ ....(*)

Now, $z^{113}=(-i\omega)^{113}=-i^{113}\cdot\omega^{113}=-i\omega^2$

and $i^{163}=i^{(4\times 40+3)}=-i$

$\therefore\quad z^{113}+i^{163}=-i(1+\omega^2)=i\omega$

$\therefore\quad (z^{113}+i^{163})^{235}=(i\omega)^{235}=i^{235}\,\omega^{235}=-i\omega=z.$

24. (A) → (Q, R, T, U), (B) → (P), (C) → S

(A) As $1, \omega, \omega^2, ..., \omega^{n-1}$ are the n, nth roots of unity,

$\therefore\quad z^n-1=(z-1)(z-\omega)(z-\omega^2)\,....\,(z-\omega^{n-1})$

$\Rightarrow\quad 2^n-1=(2-\omega)(2-\omega^2)\,....\,(2-\omega^{n-1})$

i.e., $(2-\omega)(2-\omega^2)\,....\,(2-\omega^{n-1})=2^n-1$

$={}^nC_1+{}^nC_2+...+{}^nC_n={}^nC_{n-1}+{}^nC_{n-2}+...+{}^nC_0$

Also, $2^{2n+1}=({}^{2n+1}C_0+{}^{2n+1}C_1+....+{}^{2n+1}C_n)$

$+({}^{2n+1}C_{n+1}+{}^{2n+1}C_{n+2}+...+{}^{2n+1}C_{2n+1})$

$\Rightarrow\quad 2^{2n+1}=2({}^{2n+1}C_0+{}^{2n+1}C_1+...+{}^{2n+1}C_n)$

$\Rightarrow\quad (2^n)^2={}^{2n+1}C_0+{}^{2n+1}C_1+...+{}^{2n+1}C_n$

$={}^{2n+1}C_{n+1}+{}^{2n+1}C_{n+2}+....+{}^{2n+1}C_{2n+1}$

$\Rightarrow\quad 2^n=\sqrt{{}^{2n+1}C_0+{}^{2n+1}C_1+....+{}^{2n+1}C_n}$

$=\sqrt{{}^{2n+1}C_{n+1}+{}^{2n+1}C_{n+2}+....+{}^{2n+1}C_{2n+1}}$

$\Rightarrow\quad 2^n-1=\sqrt{{}^{2n+1}C_0+{}^{2n+1}C_1+{}^{2n+1}C_2....+{}^{2n+1}C_n}-1$

$=\sqrt{{}^{2n+1}C_{n+1}+{}^{2n+1}C_{n+2}+....+{}^{2n+1}C_{2n+1}}-1$

(B) Consider, $4\left|\sum_{i=1}^{n}\dfrac{1}{z_i}\right|=4\left|\dfrac{1}{z_1}+\dfrac{1}{z_2}+....+\dfrac{1}{z_n}\right|$

$$=\left|\frac{4}{z_1}+\frac{4}{z_2}+\frac{4}{z_3}+....+\frac{4}{z_n}\right|$$

$$=\left|\frac{|z_1|^2}{z_1}+\frac{|z_2|^2}{z_2}+....+\frac{|z_n|^2}{z_n}\right| \qquad (\because |z|=2)$$

$$=|\bar{z}_1+\bar{z}_2+.....+\bar{z}_n| \qquad (\because |z^2|=z\bar{z})$$

$$=\left|\overline{z_1+z_2+z_3+....+z_n}\right|$$

$$=|z_1+z_2+...+z_n| \qquad (\because |z|=|\bar{z}|)$$

$\therefore\quad |z_1+z_2+....+z_n|-4\left|\dfrac{1}{z_1}+\dfrac{1}{z_2}+...+\dfrac{1}{z_n}\right|=0$

$\Rightarrow\quad \left|\sum_{i=1}^{n}z_i\right|-4\left|\sum_{i=1}^{n}\dfrac{1}{z_i}\right|=0=(1-x)^n$ for $x=1$

$\Rightarrow\quad {}^nC_0-{}^nC_1+{}^nC_2-{}^nC_3+...=0$

(C) Let $A=\sum_{i=0}^{n-1}\left(\dfrac{\alpha_i}{3-\alpha_i}\right)$

$$=-\sum_{i=0}^{n-1}\frac{-\alpha_i}{3-\alpha_i}=-\sum_{i=0}^{n-1}\left[\frac{(3-\alpha_i)-3}{3-\alpha_i}\right]$$

$$=-\sum_{i=0}^{n-1}1+3\sum_{i=0}^{n-1}\frac{1}{3-\alpha_i} \qquad ...(1)$$

$\because\quad z^n-1=\prod_{i=0}^{n-1}(z-\alpha_i)$

$\Rightarrow\quad \log(z^n-1)=\sum_{i=0}^{n-1}\log(z-\alpha_i)$ ...(*)

On differentiating w.r.t. z both sides of (*), we get

$$\frac{nz^{n-1}}{z^n-1}=\sum_{i=0}^{n-1}\frac{1}{z-\alpha_i}$$

Now, taking $z=3$, we get $\dfrac{n\cdot 3^{n-1}}{3^n-1}=\sum_{i=0}^{n-1}\dfrac{1}{3-\alpha_i}$

From equation (1), we have

$$A=3\sum_{i=0}^{n-1}\frac{1}{3-\alpha_i}-n=3\cdot\frac{n\cdot 3^{n-1}}{3^n-1}-n$$

$$=\frac{n\cdot 3^n-n(3^n-1)}{3^n-1}=\frac{n}{3^n-1}\quad\therefore\ \sum_{i=0}^{n-1}\frac{\alpha_i}{3-\alpha_i}=\frac{n}{3^n-1}$$

25. (28) : Given equation is $x^{2019}+\left(\dfrac{1}{3}-x\right)^{2019}=0$ ...(*)

$\Rightarrow\quad x^{2019}-\left(x-\dfrac{1}{3}\right)^{2019}=0$

$$\Rightarrow\quad x^{2019}-\left[x^{2019}-\binom{2019}{1}x^{2018}\left(\frac{1}{3}\right)+\binom{2019}{2}x^{2017}\left(\frac{1}{3}\right)^2-.....-\left(\frac{1}{3}\right)^{2019}\right]=0$$

$$\Rightarrow\quad \binom{2019}{1}x^{2018}\frac{1}{3}-\binom{2019}{2}x^{2017}\left(\frac{1}{3}\right)^2+\binom{2019}{3}x^{2016}\left(\frac{1}{3}\right)^3-......+\left(\frac{1}{3}\right)^{2019}=0$$

$\therefore\quad$ Sum of the roots of (*) taken two at a time

$$=\frac{\text{coefficient of } x^{2016}}{\text{coefficient of } x^{2018}}=\frac{2019\cdot 2018\cdot 2017}{3\cdot 2\cdot 1}\cdot\frac{1}{27}\cdot\frac{3}{2019}$$

$\therefore\ S=\left[\frac{1009\times 2017}{27}\right]=75376$, where $[\cdot]$ denotes greatest integer function.

$\therefore$ Sum of digits of $S = 7 + 5 + 3 + 7 + 6 = 28$

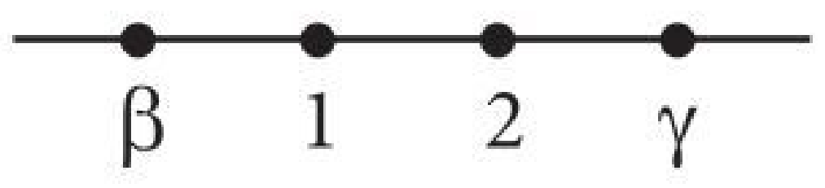
26. (390) : Let $f(x) = (\alpha - 5)x^2 - 2\alpha x + \alpha - 4\ (\alpha \neq 5)$

Let roots of $f(x) = 0$ are β, γ respectively.

As 1 and 2 lies between the roots of $f(x) = 0$, we get

$D > 0$, $(\alpha - 5)\, f(1) < 0$

and $(\alpha - 5)\, f(2) < 0$



where $f(1) = (\alpha - 5) - 2\alpha + \alpha - 4 = -9$

and $f(2) = 4(\alpha - 5) - 4\alpha + \alpha - 4 = \alpha - 24$

$D > 0 \Rightarrow 4\alpha^2 - 4(\alpha - 5)(\alpha - 4) > 0$

$\Rightarrow 9\alpha - 20 > 0 \Rightarrow \alpha \in \left(\frac{20}{9}, \infty\right)$ ...(i)

$(\alpha - 5)\, f(1) < 0 \Rightarrow (\alpha - 5)(-9) < 0$

$\Rightarrow \alpha - 5 > 0 \Rightarrow \alpha \in (5, \infty)$ ...(ii)

$(\alpha - 5)\, f(2) < 0 \Rightarrow (\alpha - 5)(\alpha - 24) < 0$

$\Rightarrow \alpha \in (5, 24)$ ...(iii)

From (i), (ii) and (iii), we get $\alpha \in (5, 24)$

$\Rightarrow \alpha = 6, 7, 8, 9, 10, 11, 12, ..., 23$

$\therefore$ Sum of all integral values of $\alpha = 6 + 7 + ... + 23$, which is an A.P. having $a = 6$, $l = 23$, $n = 18$

Let S = sum of all integral values of α

$$=\frac{n}{2}(a+l)=\frac{18}{2}(6+23) = 9 \times 29 = 3^2 \times 29^1$$

$$\therefore \text{ Sum of all divisors of } S = \frac{3^3-1}{3-1}\times\frac{29^2-1}{29-1}$$

$$=13\times\frac{840}{28}=390$$

27. (0) : Given, $f(x) = x^2 - ax + b$ and we are given a is an odd positive integer.

Let roots of $x^2 - ax + b = 0$ be x_1 and x_2 ($x_1 \neq x_2$ given)

$\therefore$ $x_1 + x_2 = a$ is an odd positive integer and so both x_1, x_2 cannot be odd.

Therefore one root must be 2 (as roots are prime numbers)

$\therefore$ $f(2) = 0 \Rightarrow 4 - 2a + b = 0 \Rightarrow 2a - b = 4$ ...(i)

And we have $a + b = 17$ ...(ii)

Solving (i) and (ii), we get $a = 7$ and $b = 10$

$\therefore$ $f(x) = x^2 - 7x + 10$

$\therefore$ $f(4) = 16 - 28 + 10 = -2$

and $\sum_{r=1}^{4} f(r)=\sum_{r=1}^{4}(r^2-7r+10)$

$$=\frac{4\cdot 5\cdot 9}{6}-\frac{7\cdot 4\cdot 5}{2}+10\times 4=0$$

$$\therefore\ f(4)\sum_{r=1}^{4} f(r)=(-2)(0)=0$$

28. (3) :
$$\left.\begin{aligned} x &= a + b\omega + c\omega^2\\ y &= a\omega + b\omega^2 + c\\ z &= a\omega^2 + b + c\omega\end{aligned}\right\}\quad ...(i)$$

On adding equations of (i), we get

$x + y + z = (1 + \omega + \omega^2)(a + b + c) = 0$

$\therefore\ x^3 + y^3 + z^3 = 3xyz \Rightarrow \frac{x^3+y^3+z^3}{xyz}=3$

$\Rightarrow \frac{x^2}{yz}+\frac{y^2}{xz}+\frac{z^2}{xy}=3$

29. (710) : Consider, $\sum_{b=1}^{6}\left\{\sin\left(\frac{2b\pi}{7}\right)-i\cos\left(\frac{2b\pi}{7}\right)\right\}$

$$=-i\sum_{b=1}^{6}\left\{\cos\left(\frac{2b\pi}{7}\right)+i\sin\left(\frac{2b\pi}{7}\right)\right\}$$

$$=-i\sum_{b=1}^{6} e^{i\left(\frac{2b\pi}{7}\right)}=-i\left[\sum_{b=0}^{6} e^{i\left(\frac{2b\pi}{7}\right)}-1\right]$$

$= -i[0 - 1] = i$ [$\because$ sum of n^{th} roots of unity = 0]

$$\therefore\ \sum_{a=1}^{20}(3a+4)\left[\sum_{b=1}^{6}\left\{\sin\left(\frac{2b\pi}{7}\right)-i\cos\left(\frac{2b\pi}{7}\right)\right\}\right]^{4a}$$

$= \sum_{a=1}^{20}(3a+4)(i)^{4a} = \sum_{a=1}^{20}(3a+4)$ ($\because i^{4a} = (i^4)^a = 1$)

$$=\frac{3\times 20\times 21}{2}+4\times 20 = 630 + 80 = 710$$

30. (–1) : $\prod_{m=1}^{4} z_m = z_1 z_2 z_3 z_4$

$$=\left(\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}\right)\left(\cos\frac{2\pi}{10}+i\sin\frac{2\pi}{10}\right)\left(\cos\frac{3\pi}{10}+i\sin\frac{3\pi}{10}\right)\left(\cos\frac{4\pi}{10}+i\sin\frac{4\pi}{10}\right)$$

$$=\cos\left(\frac{\pi}{10}+\frac{2\pi}{10}+\frac{3\pi}{10}+\frac{4\pi}{10}\right)+i\sin\left(\frac{\pi}{10}+\frac{2\pi}{10}+\frac{3\pi}{10}+\frac{4\pi}{10}\right)$$

[$\because$ $(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)$ ...... $(\cos\theta_n + i\sin\theta_n)$ $= \cos(\theta_1 + \theta_2 + ... + \theta_n) + i\sin(\theta_1 + \theta_2 + ... + \theta_n)$]

$$=\cos\left(\frac{10\pi}{10}\right)+i\sin\left(\frac{10\pi}{10}\right)=\cos\pi+i\sin\pi = -1$$

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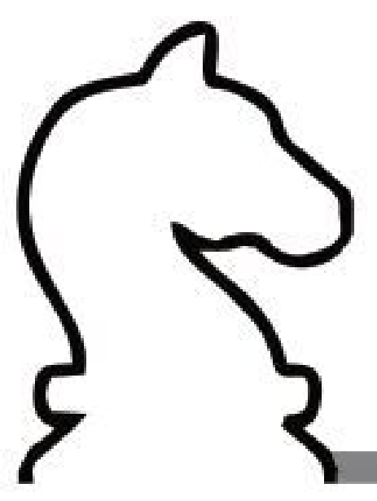
Challenging PROBLEMS ON JEE



1. If 7 divides $32^{32^{32}}$, then the remainder is

(a) 1 (b) 0 (c) 4 (d) 6

2. Let n be an odd natural number greater than 1, then the number of zeroes at the end of the sum $99^n + 1$ is

(a) 3 (b) 4

(c) 2 (d) None of these

3. If m and n are any two odd positive integers with $n < m$, then the largest positive integer which divides all numbers of the form, $m^2 - n^2$, can be

(a) 4 (b) 6 (c) 8 (d) 9

4. The value of $\sum_{k=1}^{\infty} \frac{6^k}{(3^k - 2^k)(3^{k+1} - 2^{k+1})} =$

(a) 1 (b) 2 (c) 3 (d) 4

5. If $(x^2 + 2x + 4)^n = \sum_{r=0}^{2n} a_r x^r$, then $\sum_{r=n}^{2r} \left(\frac{a_{2n-r}}{a_r} \right)$ is

(a) $2^{2n+1} - 1$ (b) $\frac{4^{n+1} - 1}{3}$

(c) $4^n - 1$ (d) $\frac{4^{n-1} + 1}{2}$

6. If $P_k = \frac{1 - x^{k+1}}{1 - x}$, then the number of terms in the product $P_1 P_2 \ldots P_n$ is

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n^2 - n}{2}$

(c) $\frac{n^2 + n - 2}{2}$ (d) $\frac{n^2 + n + 2}{2}$

7. The value of $\sum_{m=1}^{n} \left(\sum_{k=1}^{m} \left(\sum_{p=k}^{m} {}^nC_m \cdot {}^mC_p \cdot {}^pC_k \right) \right) =$

(a) $3^n - 2^n$ (b) $4^n - 3^n$

(c) $3^n + 2^n$ (d) $4^n - 1$

8. The term independent of x and y in the expansion of $[(\sqrt{x} + \frac{1}{\sqrt{x}})^2 + (\sqrt{y} + \frac{1}{\sqrt{y}})^2 + (\sqrt{xy} + \frac{1}{\sqrt{xy}})^2 - 4]^n$ is

(a) $\left(\sum_{r=0}^{n} {}^nC_r \right)^2$ (b) $\sum_{r=0}^{n} ({}^nC_r)^2$

(c) $\left(\sum_{r=0}^{n} {}^nC_r \right)^3$ (d) $\sum_{r=0}^{n} ({}^nC_r)^3$

9. If $C_0, C_1, C_2, \ldots, C_n$ denote the binomial coefficients in the expansion of $(1 + x)^n$, then $\sum_{r=0}^{n} (-1)^r \, {}^nC_r \frac{1 + r \log_e 10}{(1 + \log_e 10^n)^r}$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3

10. If $(1 + px + x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then $\sum_{r=0}^{2n} (2r + 1) a_r =$

(a) $(p + 2)^n$ (b) $(2p + 1)(p + 2)^2$

(c) $(2n + 1)(p + 2)^n$ (d) $(p + 2)^{n+1}$

11. Let $(x^3 + \alpha x^2 + 2x - 5)^{19} (x^2 + \beta x - 41)^8 (x^4 - x^3 + x - 7)^6 = x^{97} + 391x^{96} + a_{95} x^{95} + a_{94} x^{94} + \ldots + a_1 x + a_0$ be an identity, where $\alpha, \beta, a_{95}, a_{94}, \ldots, a_1, a_0$ are integers. If $\alpha + \beta < 10$, then the smallest possible value of α is

(a) 7 (b) 8 (c) 31 (d) 23

12. The number of homogeneous products of degree 3 from 4 variables is equal to

(a) 20 (b) 16 (c) 12 (d) 4

13. Consider $S = \{1, 2, 3, 4, \ldots, 10\}$. Then sum of all products of numbers by taking two or more from S is $(11! - K)$, then $\left[\frac{K}{11} \right]$ is equal to (where $[\cdot]$ is greatest integer function)

(a) 1 (b) 2 (c) 4 (d) 5

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14. If $x, y \in (0, 30)$ such that

$$\left[\frac{x}{3}\right]+\left[\frac{3x}{2}\right]+\left[\frac{y}{2}\right]+\left[\frac{3y}{4}\right]=\frac{11}{6}x+\frac{5}{4}y$$

(where $[\cdot]$ denotes the greatest integer $\le x$). Then the number of ordered pairs (x, y) is

(a) 0 (b) 2 (c) 4 (d) 28

15. Let N be a natural number if its first digit (from the left) is deleted, it gets reduced to $\left[\frac{N}{57}\right]$. The sum of all the digits of N is

(a) 15 (b) 18 (c) 24 (d) 30

16. The sum of the series $1+\frac{9}{4}+\frac{36}{9}+\frac{100}{16}+....$ up to n terms if $n = 16$ is

(a) 446 (b) 746 (c) 546 (d) 846

17. The sequence $\{x_1, x_2, ..., x_{50}\}$ has the property that for each k, x_k is k less than the sum of other 49 numbers. The value of $96x_{20}$ is

(a) 300 (b) 315 (c) 1024 (d) 0

18. If a_1 is the greatest value of $f(x)$, where $f(x)=\left(\frac{1}{2+[\sin x]}\right)$ (where $[\cdot]$ denotes greatest integer function) and $a_{n+1}=\frac{(-1)^{n+2}}{(n+1)}+a_n$, then $\lim\limits_{n\to\infty}(a_n)$ is equal to

(a) 1 (b) e^2 (c) ln2 (d) ln3

19. The sequence $\{x_k\}$ is defined by $x_{k+1}=x_k^2+x_k$ and $x_1 = 1/2$. Then $\left[\frac{1}{x_1+1}+\frac{1}{x_2+1}+...+\frac{1}{x_{100}+1}\right]$ (where $[\cdot]$ denotes the greatest integer function) is equal to

(a) 0 (b) 2 (c) 4 (d) 1

20. $\{a_n\}$ and $\{b_n\}$ be two sequences given by $a_n = (x)^{1/2^n} + (y)^{1/2^n}$ and $b_n = (x)^{1/2^n} - (y)^{1/2^n}$ for all $n \in N$, then $a_1 a_2 a_3 \ldots a_n$ is equal to

(a) $x - y$ (b) $\frac{x+y}{b_n}$

(c) $\frac{x-y}{b_n}$ (d) $\frac{xy}{b_n}$

21. The sum of the series

$$\frac{9}{5^2\cdot2\cdot1}+\frac{13}{5^3\cdot3\cdot2}+\frac{17}{5^4\cdot4\cdot3}+....\text{ upto infinity, is}$$

(a) 1 (b) 9/5 (c) 1/5 (d) 2/5

22. If $S_n=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+.....+\frac{1}{n}$ and $n > 2$, then S_n would always be

(a) more than $n(n+1)^{1/n}-n$

(b) less than $n(n + 1)^{1/n} - n$

(c) equal to $n(n+1)^{1/n}-n$

(d) greater than or equal to $\frac{n(n+1)^{1/n}}{(n+5)}$

SOLUTIONS

1. (c) : Consider, $32 = 2^5 \Rightarrow (32)^{32} = (2^5)^{32}$

$= 2^{160} = (3 - 1)^{160} = 3m + 1, \forall\ m \in N$

$\therefore\ (32)^{32^{32}}=(32)^{3m+1}=2^{5(3m+1)}$

$= 2^{3(5m + 1)}\, 2^2 = 4\cdot8^{5m + 1}$

$= 4(7 + 1)^{5m + 1} = 4(7n + 1), \forall\ n \in N = 28n + 4$

$\therefore$ When 7 divides $(32)^{32^{32}}$, we get remainder = 4

2. (c) : We have, $1 + 99^n = 1 + (100 -1)^n$

$= 1 + \{^nC_0\, 100^n - {^nC_1} \cdot 100^{n-1} + ... - {^nC_n}\}$

$= 100\{^nC_0 \cdot 100^{n-1} - {^nC_1}\cdot100^{n-2}$

$+.... - {^nC_{n-2}} \cdot 100 + {^nC_{n-1}}\}$

$= 100 \times$ integer, whose units place is different from 0

[$\because\ {^nC_{n-1}} = n$, has odd digit at unit place]

$\therefore$ There are two zeroes at the end of the sum $99^n + 1$.

3. (c) : Let $m = 2k - 1$ and $n = 2p - 1$, $p < k$

Then $m^2 - n^2 = (m + n)(m - n)$

$= (2k + 2p - 2)(2k - 2p) = 4(k + p - 1)(k - p)$

Further if k and p both are even, then $k - p$ is even but $k + p - 1$ is odd.

If k and p both are odd, then $k - p$ is even but $k + p - 1$ is odd. If one is even and other is odd, then $k - p$ is odd but $k + p - 1$ is even. Thus in every case $(k - p)(k + p - 1)$ even.

$\therefore$ $m^2 - n^2$ is divisible by $4 \times 2 = 8$. Hence, $m^2 - n^2$ is divisible by 8 or any multiple of 8. The largest integer among the given options is 8.

4. (b) : We have,

$$\frac{6^k}{(3^k-2^k)(3^{k+1}-2^{k+1})}=\frac{3^k\cdot2^k}{(3^k-2^k)(3^{k+1}-2^{k+1})}$$

$$=\frac{3^k}{3^k-2^k}-\frac{3^{k+1}}{3^{k+1}-2^{k+1}}$$

$$\therefore\quad \sum_{k=1}^{\infty}\left(\frac{3^k}{3^k-2^k}-\frac{3^{k+1}}{3^{k+1}-2^{k+1}}\right)=\frac{3}{3-2}-\operatorname*{Lt}_{n\to\infty}\frac{3^n}{3^n-2^n}$$

$$= 3 - 1 = 2$$

5. (b) : Putting $x = 2x$, given equation becomes

$$(4x^2+4x+4)^n = \sum_{r=0}^{2n} a_r 2^r x^r$$

$$\Rightarrow 4^n(x^2+x+1)^n = \sum_{r=0}^{2n} a_r 2^r x^r \quad \text{...(i)}$$

Putting $x = 1/x$, given equation becomes

$$\left[1+2x+(2x)^2\right]^n = \sum_{r=0}^{2n} a_r\, x^{2n-r} \quad \text{...(ii)}$$

Putting $2x = y$ in (ii), we get

$$(1+y+y^2)^n = \sum_{r=0}^{2n} a_r \frac{y^{2n-r}}{2^{2n-r}} \quad \text{...(iii)}$$

Equating coefficient of x^r in (i) and (iii) , we get

$$\frac{1}{4^n} a_r 2^r = a_{2n-r} 2^{-r}$$

$$\Rightarrow \quad \frac{a_{2n-r}}{a_r} = \frac{1}{4^n} 4^r$$

$$\therefore \quad \sum_{r=n}^{2n} \frac{a_{2n-r}}{a_n} = \sum_{r=n}^{2n} \frac{1}{4^n} 4^r = \left(\frac{4^{n+1}-1}{3}\right)$$

6. (d) : $P_k = \dfrac{1-x^{k+1}}{1-x}$

Now, $P_1P_2P_3.....P_n = \dfrac{(1-x^2)(1-x^3)(1-x^4).....(1-x^{n+1})}{(1-x)^n}$

$\therefore$ Number of terms = 1 + maximum power of x

$$= 1+\frac{n(n+1)}{2} = \frac{n^2+n+2}{2}$$

7. (b) : We have ,

$$\sum_{m=1}^{n} {}^nC_m\left(\sum_{k=1}^{m}\left(\sum_{p=k}^{m} \frac{m!}{p!(m-p)!}\cdot\frac{p!}{k!(p-k)!}\right)\right)$$

$$= \sum_{m=1}^{n} {}^nC_m\left(\sum_{k=1}^{m}\left(\sum_{p=k}^{m} {}^{m-k}C_{p-k}\right)\frac{m!}{k!(m-k)!}\right)$$

$$= \sum_{m=1}^{n} {}^nC_m\left(\sum_{k=1}^{m} 2^{m-k}\cdot {}^mC_k\right)$$

$$= \sum_{m=1}^{n} {}^nC_m((1+2)^m-2^m) = \sum_{m=1}^{n}\left({}^nC_m 3^m - {}^nC_m 2^m\right)$$

$= \{(1 + 3)^n - 1\} - \{(1 + 2)^n-1\} = 4^n - 3^n$

8. (d) : The given expression can be written as

$$(1+x)^n\cdot(1+y)^n\cdot\left(1+\frac{1}{xy}\right)^n$$

$\therefore$ The constant term is clearly $C_0^3 + C_1^3 + ... + C_n^3$, where $C_r = {}^nC_r$.

9. (a) : Let $\log_e 10 = x$

$$\therefore \quad \sum_{r=0}^{n}(-1)^r\ {}^nC_r \frac{1+r\cdot\log_e 10}{\left(1+\log_e 10^n\right)^r}$$

$$= \sum_{r=0}^{n}(-1)^r\ {}^nC_r \frac{1+rx}{(1+nx)^r}$$

$$= \left(1-\frac{1}{1+nx}\right)^n - \frac{nx}{1+nx}\left(1-\frac{1}{1+nx}\right)^{n-1} = 0$$

10. (c) : We have, $\displaystyle\sum_{r=0}^{2n}(2r+1)a_r$

$$= 2\sum_{r=0}^{2n} ra_r + \sum_{r=0}^{2n} a_r$$

$$= 2n(2+p)^{n-1}(p+2)+(2+p)^n$$

$$= (2n + 1)(p + 2)^n$$

11. (c) : It will be an identity even if we replace x by $\dfrac{1}{y}$ and considering numerator alone. Differentiating on both sides with respect to y, at $y = 0$ we get $19\alpha + 8\beta = 397$, $\alpha + \beta = 10 - k$ where k is positive integer. Put $\beta = 10 - \alpha - k$ in first equation, we get $11\alpha - 8k = 317$

$\therefore \quad \alpha = 31$

12. (a) : Let four variables are x, y, z and t.

So, $x^ay^bz^ct^d$ such that $a + b + c + d = 3$

$\therefore$ Number of homogeneous products $= {}^{4+3-1}C_3 = 20$

13. (d) : $(x + 1)(x + 2) .... (x + 10)$

$= x^{10} + (\Sigma 1)x^9 + (\Sigma 1\cdot 2)x^8 + ... + 1\cdot 2\cdot 3\cdot ..... \cdot 10$

Putting $x = 1$, we get

$$11! = 1+\frac{10\cdot 11}{2}+(\Sigma 1\cdot 2+\Sigma 1\cdot 2\cdot 3+...)$$

$$\Rightarrow \quad \Sigma 1\cdot 2+\Sigma 1\cdot 2\cdot 3+....... = 11!-56$$

$$\Rightarrow \quad K = 56 \ \therefore \left[\frac{K}{11}\right] = 5$$

14. (d) : From the given condition,

$\left\{\dfrac{x}{3}\right\}+\left\{\dfrac{3x}{2}\right\}+\left\{\dfrac{y}{2}\right\}+\left\{\dfrac{3y}{4}\right\}=0$, where [·] denotes fractional part function.

$\Rightarrow \quad \dfrac{x}{3}, \dfrac{3x}{2}, \dfrac{y}{2}, \dfrac{3y}{4}$ must be integers.

$\therefore \quad x = 6, 12, 18, 24$

$y = 4, 8, 12, 16, 20, 24, 28$

$\therefore$ Number of ordered pairs = 4 × 7 = 28

15. (a) : Let, $N = a_n\, a_{n-1}\, a_{n-2} \ldots.\, a_2\, a_1\, a_0$

$\Rightarrow \dfrac{N}{57} = a_{n-1} \cdot a_{n-2} \ldots.\, a_2\, a_1\, a_0$

Now, $a_n 10^n = 56(a_0 + 10a_1 + 10^2 a_2 \ldots. + 10^{n-1} a_{n-1})$

$\Rightarrow$ 56 divides $a_n 10^n$

$\Rightarrow$ $a_n = 7, n \geq 3$

$\Rightarrow$ $5^3 = a_0 + 10a_1 + 10^2 a_2$

$\therefore$ Required value of N= 7125 or 71250 or 712500, etc

$\therefore$ Sum of all the digits of $N = 15$

16. (a) : The given series can be written as

$$1^3 + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + \ldots$$

$$\therefore \quad n^{\text{th}} \text{ term, } t_n = \frac{1^3 + 2^3 + 3^3 + \ldots.. + n^3}{1+3+5+\ldots. + (2n-1)}$$

$$= \frac{n^2(n+1)^2}{4n^2} = \frac{(n+1)^2}{4}$$

$$= \frac{1}{4}(n^2 + 2n + 1)$$

$$\text{Now, } S_n = \frac{1}{4}\left[\sum_{k=1}^{n} k^2 + 2\sum_{k=1}^{n} k + n\right]$$

$$= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + n(n+1) + n\right]$$

$$\therefore \quad S_{16} = \frac{1}{4}\left[\frac{16 \cdot 17 \cdot 33}{6} + 16 \cdot 17 + 16\right]$$

$$= \frac{1}{4}[88 \times 17 + 16 \times 17 + 16] = 446$$

17. (b) : We have, $x_k + k = S - x_k$,
where $x_1 + x_2 + \ldots. + x_k = S$

$$\Rightarrow \quad 2x_k + k = S \quad \Rightarrow \quad 2(S) + \frac{50 \cdot 51}{2} = 50S$$

$\Rightarrow$ $48(S) = 25{\cdot}51$

$$\therefore \quad x_{20} = \frac{1}{2}\left(\frac{25 \cdot 51}{48} - 20\right) = \frac{315}{96}$$

18. (c) : $a_1 = 1$,

$$a_2 = 1 - \frac{1}{2},\ a_3 = 1 - \frac{1}{2} + \frac{1}{3}$$

....

....

$$\therefore \quad a_\infty = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots\ldots\ldots = \ln 2$$

19. (d) : $\dfrac{1}{x_{k+1}} = \dfrac{1}{x_k(x_k+1)} = \dfrac{1}{x_k} - \dfrac{1}{x_k+1}$

$$\Rightarrow \quad \frac{1}{x_k+1} = \frac{1}{x_k} - \frac{1}{x_{k+1}}$$

$$\therefore \quad \frac{1}{x_1+1} + \frac{1}{x_2+1} + \ldots + \frac{1}{x_{100}+1} = \frac{1}{x_1} - \frac{1}{x_{101}}$$

As $0 < \dfrac{1}{x_{101}} < 1$

$$\therefore \quad \left[\frac{1}{x_1+1} + \frac{1}{x_2+1} + \ldots + \frac{1}{x_{100}+1}\right] = 1$$

20. (c) : $a_1\ a_2\ \ldots..\ a_n = b_n \dfrac{a_1 a_2 \ldots a_n}{b_n}$

$$= a_n\ b_n \frac{(a_1 a_2 \ldots a_{n-1})}{b_n}$$

$$= \left(x^{\frac{1}{2^{n-1}}} - y^{\frac{1}{2^{n-1}}}\right) \frac{(a_1\, a_2 \ldots a_{n-1})}{b_n}$$

$$= a_{n-1} b_{n-1} \frac{(a_1 a_2 \ldots a_{n-2})}{b_n}$$

....

....

$$= \frac{a_1 b_1}{b_n} = \frac{(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})}{b_n} = \frac{x-y}{b_n}$$

21. (c) : r^{th} term, $t_r = \dfrac{4r+1}{5^r r(r-1)}, r \geq 2$

$$= \frac{5r - (r-1)}{5^r r(r-1)} = \frac{1}{5^{r-1}(r-1)} - \frac{1}{5^r r}$$

$$\therefore \quad \sum_{r=2}^{\infty} t_r$$

$$= \left(\left(\frac{1}{5^1 \cdot 1} - \frac{1}{5^2 \cdot 2}\right) + \left(\frac{1}{5^2 \cdot 2} - \frac{1}{5^3 \cdot 3}\right) + \left(\frac{1}{5^3 \cdot 3} - \frac{1}{5^4 \cdot 4}\right) + \ldots\infty\right)$$

$$= \frac{1}{5}$$

22. (a) : Using A.M.-G.M. inequality, we have

$$\frac{(1+1) + \left(1 + \frac{1}{2}\right) + \left(1 + \frac{1}{3}\right) + \left(1 + \frac{1}{4}\right) + \ldots.. + \left(1 + \frac{1}{n}\right)}{n}$$

$$> \left(2 \times \frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \ldots.. \times \frac{n+1}{n}\right)^{1/n}$$

$$\Rightarrow \quad \frac{n + S_n}{n} > (n+1)^{1/n}$$

$\Rightarrow$ $S_n > n(n+1)^{1/n} - n$

JEE WORK OUTS

One or More Than One Option(s) Correct Type

1. If the angles of a triangle ABC satisfy the equation $81^{\sin^2 x} + 81^{\cos^2 x} = 30$, then the triangle can not be

(a) equilateral (b) isosceles
(c) obtuse angled (d) right angled

2. Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then

(a) $A^{-n} = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix} \forall n \in N$ (b) $\lim\limits_{n\to\infty} \dfrac{A^{-n}}{n} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$

(c) $\lim\limits_{n\to\infty} \dfrac{A^{-n}}{n^2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) none of these

3. A zoo has 20 zebras, 12 giraffes, 11 lions and 3 tigers. The number of ways a tourist can visit these animals so that he must see at least one tiger, is

(a) $21 \times 13 \times 12 \times 3$ (b) 7×2^{43}
(c) $7 \times 21 \times 13 \times 12$ (d) 6×2^{43}

4. The co-ordinates of the point of intersection of $\dfrac{x-1}{1} = \dfrac{y+3}{3} = \dfrac{z-2}{2}$ with plane $3x - 2y + z = 7$, is

(a) (10, 5, 8) (b) (5, 10, 8)
(c) (5, 9, 10) (d) None of these

5. The largest value of non-negative integer 'a' for which $\lim\limits_{x\to 1} \left(\dfrac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right)^{\frac{1-x}{1-\sqrt{x}}} = \dfrac{1}{4}$, is

(a) 2 (b) 1 (c) 0 (d) –1

6. If all the three roots of $az^3 + bz^2 + cz + d = 0$ have negative real parts ($a, b, c \in R$), then

(a) $ab > 0$ (b) $bc > 0$
(c) $ad > 0$ (d) $bc - ad > 0$

7. How many numbers between 5000 and 10000 can be formed using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 each digit appearing not more than once in each number?

(a) $5 \times {}^8P_3$ (b) $5 \times {}^8C_8$
(c) $5! \times {}^8P_3$ (d) $5 \times {}^8C_3 \times 3!$

8. If $f(x) = \lim\limits_{n\to\infty} \dfrac{x}{x^{2n} + 1}$, then

(a) $f(1^+) + f(1^-) = 0$

(b) $f(1^+) + f(1^-) + f(1) = 3/2$

(c) $f(-1^+) + f(-1^-) = -1$

(d) $f(1^+) + f(-1^-) = 0$

9. The solution of $\dfrac{dy}{dx} = \dfrac{ax+h}{by+k}$ represents a parabola if

(a) $a = -2, b = 0$ (b) $a = -2, b = 2$
(c) $a = 0, b = 2$ (d) $a = 0, b = 0$

10. Let $f : (-1, 1) \to R$ be such that $f(\cos 4\theta) = \dfrac{2}{2 - \sec^2\theta}$ for $\theta \in \left(0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$. Then the value(s) of $f\left(\dfrac{1}{3}\right)$ is /are

(a) $1 - \sqrt{\dfrac{3}{2}}$ (b) $1 + \sqrt{\dfrac{3}{2}}$ (c) $1 - \sqrt{\dfrac{2}{3}}$ (d) $1 + \sqrt{\dfrac{2}{3}}$

11. Which of the following is true for any ΔABC ?

(a) $\sin A \sin B \sin C \le \dfrac{3\sqrt{3}}{8}$

(b) $\sin^2 A + \sin^2 B + \sin^2 C \le \dfrac{9}{4}$

(c) $\sin A \sin B \sin C$ is always positive
(d) $\sin^2 A + \sin^2 B \le 1 + \cos C$

12. Let $S = \left\{ x \in (-\pi, \pi) : x \ne 0, \pm \frac{\pi}{2} \right\}$. The sum of all distinct solutions of the equation $\sqrt{3}\ \sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to

(a) $\frac{-7\pi}{9}$ (b) $\frac{-2\pi}{9}$ (c) 0 (d) $\frac{5\pi}{9}$

13. If the area enclosed by the parabola $y = 1 + x^2$ and a normal drawn to it with gradient -1, is A then $3A$ is
(a) 2 (b) 3 (c) 4 (d) 5

14. If $2 \times 2^2 + 3 \times 2^3 + 4 \times 2^4 + \dots + n \times 2^n = 2^{n+10}$, then the value of n is
(a) 513 (b) 512 (c) 511 (d) 520

15. In a ΔABC, $\frac{a}{b} = 2 + \sqrt{3}$ and $\angle C = 60°$. Then the ordered pair $(\angle A, \angle B)$ is equal to
(a) (15°, 105°) (b) (105°, 15°)
(c) (45°, 75°) (d) (75°, 45°)

16. For which values of 'a' will the function $f(x) = x^4 + ax^3 + \frac{3x^2}{2} + 1$ be concave upward along the entire real line ?
(a) $a \in [0, \infty)$ (b) $a \in [-2, \infty)$
(c) $a \in [-2, 2]$ (d) $a \in (0, \infty)$

17. The number of solutions for the equation

$$2\sin^{-1}\sqrt{x^2 - x + 1} + \cos^{-1}\sqrt{x^2 - x} = \frac{3\pi}{2} \text{ is}$$

(a) 1 (b) 2
(c) 3 (d) infinite

18. Let [·] denote the greatest integer function and $f(x) = [\tan^2 x]$. Then which of the following is not true?
(a) $\lim_{x \to 0} f(x)$ does not exist
(b) $f(x)$ is continuous at $x = 0$
(c) $f(x)$ is not differentiable at $x = 0$
(d) none of these

19. If A and B are two events such that $P(A) = 1/2$ and $P(B) = 2/3$, then
(a) $P(A \cup B) \ge 2/3$
(b) $P(A \cap B') \le 1/3$
(c) $1/6 \le P(A \cap B) \le 1/2$
(d) $1/6 \le P(A' \cap B) \le 1/2$

20. If N is the number of positive integral solutions of $x_1 x_2 x_3 x_4 = 770$, then
(a) N is divisible by 4 distinct primes
(b) N is a perfect square
(c) N is a perfect 4th power
(d) N is a perfect 8th power

Comprehension Type

Paragraph for Q. No. 21 to 23

$f(x) = x^3 - 9x^2 + 24x + c = 0$ has three real and distinct roots α, β, and γ.

21. Possible values of c are
(a) (–20, –16) (b) (–20, –18)
(c) (–18, –16) (d) none of these

22. If $[\alpha] + [\beta] + [\gamma] = 8$, then the values of c, where [·] represents the greatest integer function, are
(a) (–20, –16) (b) (–20, –18)
(c) (–18, –16) (d) none of these

23. If $[\alpha] + [\beta] + [\gamma] = 7$, then the values of c, where [·] represents the greatest integer function, are
(a) (–20, –16) (b) (–20, –18)
(c) (–18, –16) (d) none of these

Paragraph for Q. No. 24 and 25

Consider the sequence of integers in the form of group (1), (2, 2), (3, 3, 3), ..., (n, n, ..., n) n times

24. The sum of first 1100 terms is
(a) 31511 (b) 893 (c) 34404 (d) 35720

25. The sum of the remaining terms in the group after 1100 term in which 1100 term lies is
(a) 1316 (b) 1216 (c) 11316 (d) 11216

Paragraph for Q. No. 26 to 28

In a G.P., the sum of the first and last term is 66, the product of the second and the last before one is 128 and the sum of the terms is 126.

26. If an increasing G.P. is considered, then the number of terms in G.P. is
(a) 9 (b) 8 (c) 12 (d) 6

27. If the decreasing G.P. is considered, then the sum of infinite terms is
(a) 64 (b) 128 (c) 256 (d) 729

28. In any case, the difference of the least and greatest terms is
(a) 78 (b) 135
(c) 126 (d) none of these

Matrix Match Type

29. Match the following :

Column-I		Column-II	
(A)	If $L=\lim_{x\to -1}\frac{\sqrt[3]{7-x}-2}{x+1}$, then $12L=$	(P)	-2
(B)	If $L=\lim_{x\to \pi/4}\frac{\tan^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$, then $-(L/4)=$	(Q)	2
(C)	If $L=\lim_{x\to 1}\frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}$, then $20L=$	(R)	1
(D)	If $L=\lim_{x\to\infty}\frac{\log x^n-[x]}{[x]}$, where $n\in N$ ($[x]$ denotes greatest integer function), then $-2L=$	(S)	-1

30. Match the following :

Column-I		Column-II	
(A)	The area of the triangle whose vertices are the points, with rectangular cartesian coordinates (1, 2, 3), (–2, 1, –4), (3, 4, –2) is	(P)	0
(B)	The value of $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})+(\vec{b}\times\vec{c})\cdot(\vec{a}\times d)+(\vec{c}\times\vec{a})\cdot(\vec{b}\times\vec{d})$ is	(Q)	$\frac{9}{2}$
(C)	If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then value of k is	(R)	$\frac{\sqrt{1218}}{8}$
(D)	$\vec{a}=2\hat{i}+3\hat{j}-\hat{k}, \vec{b}=-\hat{i}+2\hat{j}-4\hat{k}$, $\vec{c}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{d}=3\hat{i}+2\hat{j}+\hat{k}$ then $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=$	(S)	21

31. Match the following :

Column-I		Column-II	
(A)	$\cot 82\frac{1}{2}^\circ =$	(P)	$\cot 22\frac{1}{2}^\circ$
(B)	$\cot 15^\circ =$	(Q)	$\sqrt{2}+1$
(C)	$\tan 82\frac{1}{2}^\circ =$	(R)	$2+\sqrt{3}$
(D)	$\tan 67\frac{1}{2}^\circ =$	(S)	$(\sqrt{3}+\sqrt{2})(\sqrt{2}+1)$
		(T)	$\cot 7\frac{1}{2}^\circ$
		(U)	$(\sqrt{3}-\sqrt{2})(\sqrt{2}-1)$

Numerical Answer Type

32. The sum of two digit even numbers which do not end with zero is ________.

33. If $\alpha = 2\sin^{-1}(2/3)$ and $\beta = 2\tan^{-1}9$, then $80\operatorname{cosec}^2\alpha + 81\operatorname{cosec}^2\beta$ is equal to ________.

34. The digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 are written in random order to form a nine digit number. Let p be the probability that this number is divisible by 36, the value of $9p$ is ________.

35. The probability of at least one double-six being thrown in n throws with two ordinary dice is greater than 99 percent. Calculate the least numerical value of n________. [Take $\log_{10}36 = 1.5563$ and $\log_{10}35 = 1.5441$]

36. The chord of the curve $y = -a^2x^2 + 5ax - 4$ touches the curve $y=\frac{1}{1-x}$ at the point $x = 2$ and is bisected by that point. Then the value of a is ________.

37. The number of values of θ in $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$, is __________.

38. In a triangle, L and M are midpoints of the sides AB and AC respectively. Let 'O' is the circumcentre of the ΔABC, then $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta OLM)}\cot B\cot C$ equals ________.

39. The number of solutions of the equation $\tan^2x + \cot^2x + \operatorname{cosec}^2x - 4\cos^2x = 2$ in $[0, \pi]$ is ________ .

40. In a triangle ABC, $a = 7$, $b = 6$ and $\cos(A-B)=\frac{251}{256}$, then the third side of the triangle must be ________.

SOLUTIONS

1. (d): Let $81^{\sin^2 x} = y$, then $81^{1-\cos^2 x} = y$

$\Rightarrow \quad 81^{\cos^2 x} = 81y^{-1}$ and the given equation can be written as $y + 81y^{-1} = 30$

$\Rightarrow \quad y^2 - 30y + 81 = 0 \quad \Rightarrow \quad y = 3 \quad$ or $\quad y = 27$

When, $y = 3 \quad \Rightarrow \quad 81^{\sin^2 x} = 3$

$\Rightarrow \quad 3^{4\sin^2 x} = 3 \quad \Rightarrow \quad \sin x = \pm 1/2$

$\Rightarrow \quad x = 30°$ or $150°$

When $y = 27 \quad \Rightarrow \quad 3^{4\sin^2 x} = 3^3 \quad \Rightarrow \quad \sin x = \pm\dfrac{\sqrt{3}}{2}$

$\Rightarrow x = 60° \quad$ or $\quad 120°$

$\therefore$ The possible values of the angles A, B, C are 30° or 60° or 120° or 150°, so we get the set of angles of the triangle as 30°, 30°, 120° or 60°, 60°, 60° and thus the triangle cannot be right angled.

2. (a, b, c) : $A^2 = A \cdot A = \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\3 & 1\end{bmatrix}$ and so on

$\therefore \quad A^n = \begin{bmatrix}1 & 0\\n & 1\end{bmatrix}$ and $A^{-n} = \begin{bmatrix}1 & 0\\-n & 1\end{bmatrix}$

and $\lim\limits_{n\to\infty} \dfrac{A^{-n}}{n} = \begin{bmatrix}\lim\limits_{n\to\infty}\dfrac{1}{n} & 0\\ \lim\limits_{n\to\infty}\dfrac{-n}{n} & \lim\limits_{n\to\infty}\dfrac{1}{n}\end{bmatrix} = \begin{bmatrix}0 & 0\\-1 & 0\end{bmatrix}$

and $\lim\limits_{n\to\infty} \dfrac{A^{-n}}{n^2} = \begin{bmatrix}\lim\limits_{n\to\infty}\dfrac{1}{n^2} & 0\\ \lim\limits_{n\to\infty}\dfrac{-1}{n} & \lim\limits_{n\to\infty}\dfrac{1}{n^2}\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

3. (b): Number of ways in which the tourist can visit 20 zebras (he can visit with no zebra or 1 zebra or ... 20th zebra)

$= {}^{20}C_0 + {}^{20}C_1 + \ldots + {}^{20}C_{20} = 2^{20}$

Similarly, the number of ways to visit 12 giraffes $= 2^{12}$

Also, number of ways to visit 11 lions $= 2^{11}$

Now, with restrictions on tigers, number of ways

$= ({}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3 - {}^3C_0)$

$= 2^3 - 1 = 7$

$\therefore$ Required number of ways $= 2^{20} \times 2^{12} \times 2^{11} \times 7$

$= 7 \times 2^{43}$

4. (c) : Let $\dfrac{x-1}{1} = \dfrac{y+3}{3} = \dfrac{z-2}{2} = \lambda(\text{say})$

$\therefore \quad (x, y, z) = (\lambda + 1, 3\lambda - 3, 2\lambda + 2)$

Using this in equation of plane, we get

$\therefore \quad 3(\lambda + 1) - 2(3\lambda - 3) + (2\lambda + 2) - 7 = 0$

$\Rightarrow \quad \lambda = 4$

$\therefore \quad (x, y, z) = (5, 9, 10)$ is the point of intersection of line and plane.

5. (c) : $\lim\limits_{x\to 1}\left(\dfrac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1}\right)^{\frac{1-x}{1-\sqrt{x}}} = \dfrac{1}{4}$

$\Rightarrow \quad \lim\limits_{x\to 1}\left(\dfrac{-a(x-1) + \sin(x-1)}{x + \sin(x-1) - 1}\right)^{1+\sqrt{x}} = \dfrac{1}{4}$

$\Rightarrow \quad \lim\limits_{x\to 1}\left(\dfrac{-a + \dfrac{\sin(x-1)}{(x-1)}}{1 + \dfrac{\sin(x-1)}{x-1}}\right)^{1+\sqrt{x}} = \dfrac{1}{4}$

$\Rightarrow \quad \left(\dfrac{-a+1}{2}\right)^{1+1} = \left(\dfrac{1}{2}\right)^2$

$\therefore \quad 1 - a = \pm 1 \Rightarrow a = 0, a = 2$

For $a = 2$, exponent base is negative & rejected

$\therefore \quad a = 0$ is the only value given in options, fact is that 0 is neither non negative integer nor positive integer.

6. (a, b, c): Let $z_1 = x_1$; $z_2, z_3 = x_2 \pm iy_2$

$\Rightarrow \quad z_1 + z_2 + z_3 = -\dfrac{b}{a}$

$\Rightarrow \quad x_1 + 2x_2 = -\dfrac{b}{a} < 0 \quad \Rightarrow \quad ab > 0$

Also, $z_1z_2z_3 = x_1[x_2^2 + y_2^2] = -\dfrac{d}{a} < 0 \quad \Rightarrow \quad ad > 0$

Also, $z_1z_2 + z_2z_3 + z_1z_3 = \dfrac{c}{a}$

$\Rightarrow \quad x_1(x_2 + iy_2) + x_1(x_2 - iy_2) + x_2^2 + y_2^2 = \dfrac{c}{a}$

$\Rightarrow \quad 2x_1x_2 + x_2^2 + y_2^2 = \dfrac{c}{a} > 0$

$\Rightarrow \quad \dfrac{b}{a}\dfrac{c}{a} > 0 \Rightarrow bc > 0$

7. (a, d): We are to form 4-digit numbers with either 5 or 6 or 7 or 8 or 9 at the thousand's place.

4-digit number : $\begin{array}{cccc}\times & \times & \times & \times\\ \downarrow & \downarrow & \downarrow & \downarrow\\ 1 & 2 & 3 & 4\end{array}$

No. of ways of filling 1st place = 5

After filling 1st place, we have 8 digits to be used.

No. of ways of filling 2nd, 3rd, 4th places $= {}^8P_3$

$\therefore \quad$ Total no. of numbers formed $= 5 \times {}^8P_3$.

CONCEPT
MAP

LIMITS

Class XI

Some Standard Limits

- $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$, n being a positive integer
- $\lim_{x\to 0}\frac{\sin x}{x}=1$
- $\lim_{x\to a}\frac{\sin(x-a)}{x-a}=1$
- $\lim_{x\to 0}\cos x=1$
- $\lim_{x\to 0}\frac{\tan x}{x}=1$
- $\lim_{x\to a}\frac{\tan(x-a)}{x-a}=1$
- $\lim_{x\to 0}\frac{1-\cos x}{x}=0$

Limits of Trigonometric Functions

Let f and g be two real valued functions with the same domain such that $f(x)\le g(x)$ for all x in the domain of f and g. For some a, if both $\lim_{x\to a}f(x)$ and $\lim_{x\to a}g(x)$ exist, then $\lim_{x\to a}f(x)\le\lim_{x\to a}g(x)$.

Sandwich Theorem : Let f, g and h be real functions such that $f(x)\le g(x)\le h(x)$ for all $x\in\{\text{dom } f(x)\cap\text{dom } g(x)\cap\text{dom } h(x)\}$. For some real number a, if $\lim_{x\to a}f(x)=l=\lim_{x\to a}h(x)$, then $\lim_{x\to a}g(x)=l$.

Algebra of Limits

Let $\lim_{x\to a}f(x)=l$, $\lim_{x\to a}g(x)=m$. Then

- $\lim_{x\to a}(f(x)\pm g(x))=l\pm m$
- $\lim_{x\to a}kf(x)=kl$
- $\lim_{x\to a}f(x)g(x)=lm$
- $\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{l}{m}, m\ne 0$
- $\lim_{x\to a}|f(x)|=|l|$
- $\lim_{x\to a}f(x)^{g(x)}=l^m$
- $\lim_{x\to a}e^{f(x)}=e^l$
- $\lim_{x\to a}(\ln f(x))=\ln l, l>0$

L' Hospital's Rule

Let $f(a)=0$, $g(a)=0$ and $f(x)$, $g(x)$ are differentiable functions with derivatives $f'(x)$, $g'(x)$, then $\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f(x)-f(a)}{g(x)-g(a)}$

$$=\lim_{x\to a}\frac{\frac{f(x)-f(a)}{x-a}}{\frac{g(x)-g(a)}{x-a}}=\frac{\lim_{x\to a}\frac{f(x)-f(a)}{x-a}}{\lim_{x\to a}\frac{g(x)-g(a)}{x-a}}=\frac{f'(a)}{g'(a)}$$

or $\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{f'(a)}{g'(a)}=\frac{f''(a)}{g''(a)}$ if $f'(a)=g'(a)=0$ and so on.

Definition

Limit of a function $y=f(x)$ exists iff L.H.L = R.H.L. = $f(a)$ where L.H.L = $\lim_{x\to a^-}f(x)$ means limit approaching curve at $x=a$ to the left of a and R.H.L = $\lim_{x\to a^+}f(x)$ means limit approaching curve at $x=a$ to the right of a.

Limit of a function at a point

Limit of a function at $x=a$ is the value of $f(x)$ at $x=a$ *i.e.*, $\lim_{x\to a}f(x)$.

Limit of a function at an interval

Let $f:R\to R$ be a function and $a\in R$, then we say that $\lim_{x\to a}f(x)=l$ if $|f(x)-l|\to 0$ as $|x-a|\to 0$, where l is some real number.

Evaluation of Limits by Series

Sometimes, $\lim_{x\to a}f(x)$ can be evaluated by using the following series:

- $e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots$
- $a^x=1+(\ln a)\frac{x}{1!}+(\ln a)^2\frac{x^2}{2!}+(\ln a)^3\frac{x^3}{3!}+\ldots$
- $\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots,-1<x<1$
- $\sin x=\frac{x}{1!}-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots$
- $\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots$
- $\tan x=x+\frac{x^3}{3}+\frac{2}{15}x^5+\ldots$
- $x\cot x=1-\frac{x^2}{3}-\frac{x^4}{45}-\ldots.$
- $\sec x=1+\frac{x^2}{2}+\frac{5}{24}x^4+\ldots$
- $x\,\text{cosec}\,x=1+\frac{x^2}{6}+\frac{7x^4}{360}+\ldots$
- $\sin^{-1}x=x+\frac{1}{2}\cdot\frac{x^3}{3}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{x^5}{5}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{x^7}{7}+\ldots$
- $\tan^{-1}x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\ldots$
- $(1+x)^{1/x}=e^{\left(1-\frac{x}{2}+\frac{11x^2}{24}-\ldots\right)}$

LINES

Class XII

CONCEPT
MAP

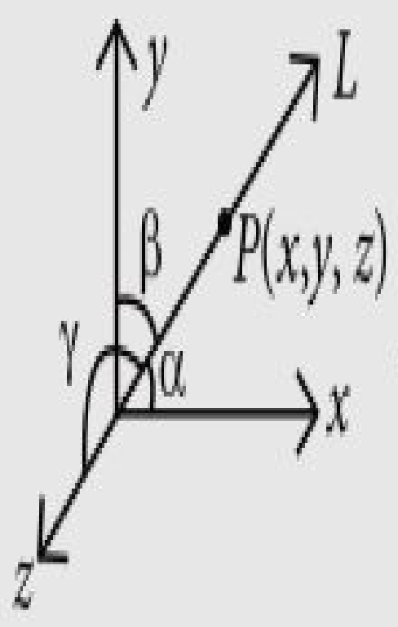
Direction Cosines and Direction Ratios

- **Direction cosines :** If a line L makes an angle α, β, γ $(0\le\alpha,\beta,\gamma\le\pi)$ with the positive direction of x, y, z axes respectively then $(l, m, n)=(\cos\alpha, \cos\beta, \cos\gamma)$ are the direction cosines of the line L and $l^2+m^2+n^2=1$.

$$\cos\alpha=\frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$\cos\beta=\frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$\cos\gamma=\frac{z}{\sqrt{x^2+y^2+z^2}}$$



D.C.s of axes:
x-axis : (1, 0, 0)
y-axis : (0, 1, 0)
z-axis : (0, 0, 1)

- **Direction ratios :** Any three numbers proportional to (l, m, n) are called direction ratios. We denote them by (a, b, c).

$\therefore\ \frac{l}{a}=\frac{m}{b}=\frac{n}{c}=k$

Shortest Distance between Two Lines

- **Vector form :** Let l_1 and l_2 be two skew lines whose equations are $\vec r=\vec a_1+\lambda\vec b_1$ and $\vec r=\vec a_2+\mu\vec b_2$ respectively. Let $\overrightarrow{PQ}$ be the shortest distance vector between l_1 and l_2. Then, $PQ=\left|\frac{(\vec b_1\times\vec b_2)\cdot(\vec a_2-\vec a_1)}{|\vec b_1\times\vec b_2|}\right|$

  If lines are parallel, then

  $\vec b_1=\vec b_2=\vec b$ and $d=\left|\frac{\vec b\times(\vec a_2-\vec a_1)}{|\vec b|}\right|$.

- **Cartesian form :** Let two skew lines be

  $l_1:\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and

  $l_2:\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$.

  The shortest distance between l_1 and l_2 is given by

$$\left|\frac{\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}\right|.$$

Equation of Line in Different Forms

- **Equation of a line in space :**

  Vector form : $\vec r=\vec a+\lambda\vec b$, $\lambda\in R$, where $A(\vec a)$ is passing point and $\vec b$ is the parallel vector.

  Cartesian form : $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$, where (x_1, y_1, z_1) is passing point and (a_1, b_1, c_1) are direction ratios of the line.

- **Line passing through two points :**

  Vector form : $\vec r=\vec a+\lambda(\vec b-\vec a)$, $\lambda\in R$, where $A(\vec a)$ and $B(\vec b)$ are the passing points.

  Cartesian form : $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$, where $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ are the passing points.

Angle Between Two Lines

- **Vector form :** Let $\vec r_1=\vec a_1+\lambda\vec b_1$ & $\vec r_2=\vec a_2+\mu\vec b_2$ be the two lines, then $\cos\theta=\left|\frac{\vec b_1\cdot\vec b_2}{|\vec b_1||\vec b_2|}\right|$.
- **Cartesian form :** Let $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ be the two lines then

  $\cos\theta=\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$.

Coplanarity of Two Lines

- **Vector form :** Two lines $\vec r_1=a_1+\lambda_1\vec n_1$, $\vec r_2=\vec a_2+\lambda_2\vec n_2$ are coplanar if $(\vec a_2-\vec a_1)\cdot(\vec n_1\times\vec n_2)=0$.
- **Cartesian form :** Two lines $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$ are coplanar if

$$\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ l_1 & m_1 & n_1\\ l_2 & m_2 & n_2\end{vmatrix}=0.$$

8. (b, c, d) : $f(x)=\lim\limits_{n\to\infty}\dfrac{x}{x^{2n}+1}=\begin{cases} x, & x^2<1\\ 0, & x^2>1\\ \dfrac{1}{2}, & x=1\\ \dfrac{-1}{2}, & x=-1\end{cases}$

$\Rightarrow\ f(1^+)=f(-1^-)=0;\ f(1^-)=1, f(-1^+)=-1,\ f(1)=\dfrac{1}{2}$

9. (a, c) : We have $\int(by+k)dy=\int(ax+h)dx$

$\Rightarrow\ \dfrac{by^2}{2}+ky=\dfrac{ax^2}{2}+hx+c$

Clearly for $a=-2$, $b=0$ and for $a=0$, $b=2$ it represents a parabola ($\because$ $y=ax^2+bx+c$ and $x=ay^2+by+c$ represents a parabola.)

10. (a,b) : $f(\cos 4\theta)=\dfrac{2}{2-\sec^2\theta}$

For $\theta\in\left(0,\dfrac{\pi}{4}\right)\cup\left(\dfrac{\pi}{4},\dfrac{\pi}{2}\right)$

Let $\cos 4\theta=\dfrac{1}{3}$

$\Rightarrow\ \cos 2\theta=\pm\sqrt{\dfrac{1+\cos 4\theta}{2}}=\pm\sqrt{\dfrac{2}{3}}$

$\therefore\ f\left(\dfrac{1}{3}\right)=\dfrac{2}{2-\sec^2\theta}=\dfrac{2\cos^2\theta}{2\cos^2\theta-1}=1+\dfrac{1}{\cos 2\theta}$

$\therefore\ f\left(\dfrac{1}{3}\right)=1-\sqrt{\dfrac{3}{2}}$ or $1+\sqrt{\dfrac{3}{2}}$

11. (a,b,c,d) : $\sin^2A+\sin^2B+\sin^2C$

$=\dfrac{1}{2}(1-\cos 2A)+\dfrac{1}{2}(1-\cos 2B)+\sin^2 C$

$=1-\dfrac{1}{2}(\cos 2A+\cos 2B)+\sin^2 C$

$=1-\dfrac{1}{2}[2\cos(A+B)\cdot\cos(A-B)]+\sin^2 C$

$=1+\cos C\cdot\cos(A-B)+1-\cos^2 C$

$=2-\cos^2 C+\cos C\cdot\cos(A-B)$

$\therefore\ \sin^2A+\sin^2B+\sin^2C\le 2-\cos^2C+\cos C$ ...(i)

$=2-(\cos^2C-\cos C)$

$=2-\left(\cos^2 C-\cos C+\dfrac{1}{4}\right)+\dfrac{1}{4}=\dfrac{9}{4}-\left(\cos C-\dfrac{1}{2}\right)^2$

$\therefore\ \sin^2A+\sin^2B+\sin^2C\le\dfrac{9}{4}$ ...(ii)

Now, for positive quantities A.M. ≥ G.M.

$\therefore\ \dfrac{\sin^2A+\sin^2B+\sin^2C}{3}\ge\sqrt[3]{\sin^2A\sin^2B\sin^2C}$

$\therefore$ From eq.(ii),

$\dfrac{9/4}{3}\ge\dfrac{\sin^2A+\sin^2B+\sin^2C}{3}\ge(\sin A\,\sin B\,\sin C)^{2/3}$

$\Rightarrow\dfrac{3}{4}\ge(\sin A\sin B\sin C)^{2/3}$

or $\sin A\,\sin B\,\sin C\le\left(\dfrac{3}{4}\right)^{3/2}=\dfrac{3\sqrt{3}}{8}$

Also, in ΔABC, $\sin A>0$, $\sin B>0$, $\sin C>0$ and from eq.(i), $\sin^2A+\sin^2B\le 1+\cos C$

12. (c) : We have, $\sqrt{3}\sec x+\operatorname{cosec} x=2(\cot x-\tan x)$

$\Rightarrow\sqrt{3}\sin x+\cos x=2\cos 2x$

$\Rightarrow\ \dfrac{\sqrt{3}}{2}\sin x+\dfrac{1}{2}\cos x=\cos 2x\ \Rightarrow\ \cos\left(x-\dfrac{\pi}{3}\right)=\cos 2x$

We have, $2x\pm\left(x-\dfrac{\pi}{3}\right)=2k\pi, k\in Z$

Then $x=(6k-1)\dfrac{\pi}{3}$ or $(6k+1)\dfrac{\pi}{9}, k\in Z$

The values of x in $(-\pi,\pi)$ are $-\dfrac{\pi}{3},\dfrac{\pi}{9},\dfrac{7\pi}{9},-\dfrac{5\pi}{9}$

$\therefore$ Required sum = 0

13. (c) : Given, $y=1+x^2$ ...(i)

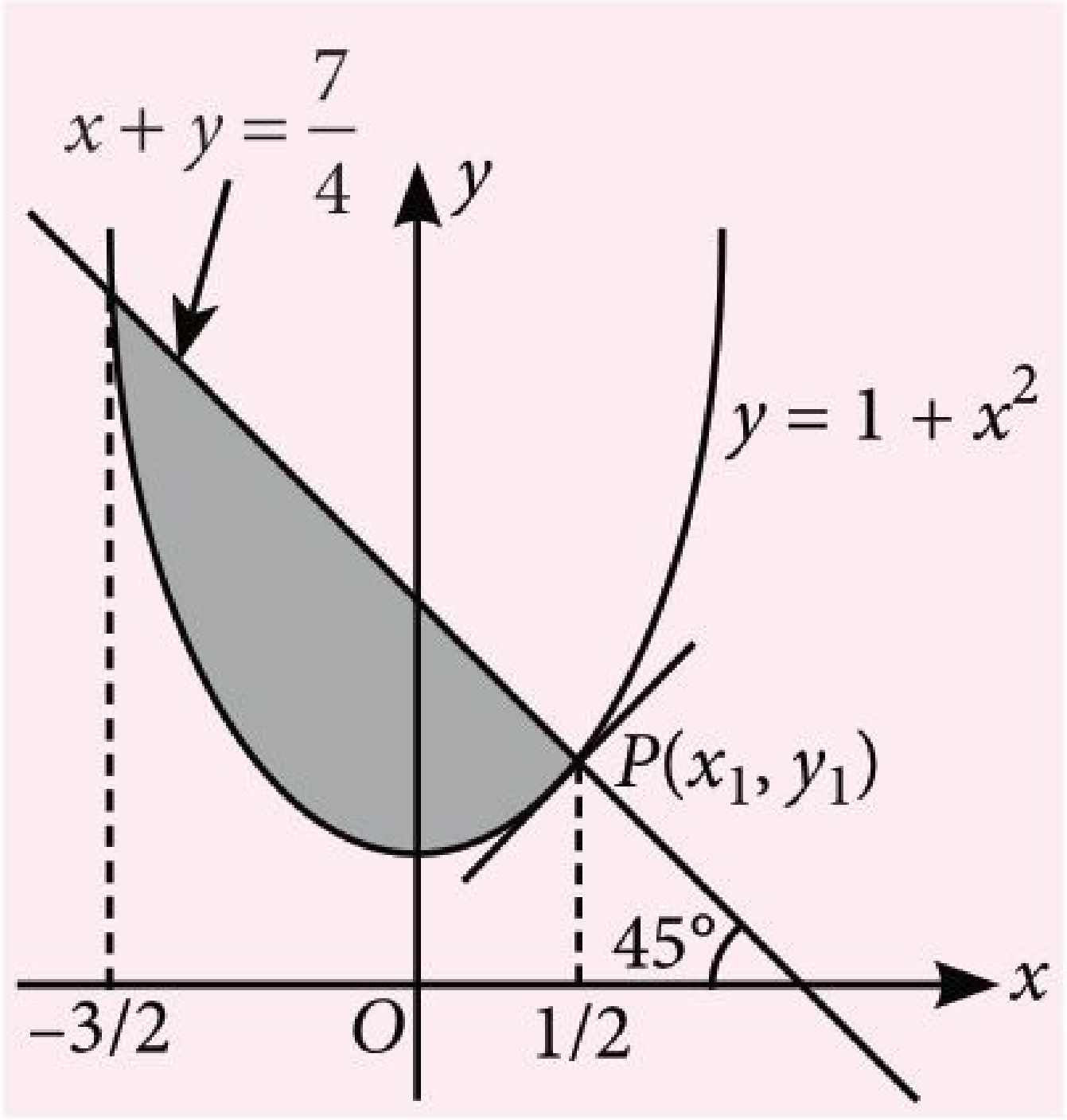
$\Rightarrow\ \dfrac{dy}{dx}=2x$

Hence, $m=-\dfrac{1}{2x_1}$ (where m = slope of normal at (x_1, y_1))

$\therefore\ x_1=\dfrac{-1}{2m}=\dfrac{1}{2}$; $y_1=1+\dfrac{1}{4m^2}=\dfrac{5}{4}$ [$\because$ $m=-1$]

Equation of normal is $y-\dfrac{5}{4}=-\left(x-\dfrac{1}{2}\right)$

$\Rightarrow\ x+y=\dfrac{5}{4}+\dfrac{1}{2}=\dfrac{7}{4}$ ...(ii)



$A=\displaystyle\int_{x_1}^{x_2}\left(\left(\frac{7}{4}-x\right)-(1+x^2)\right)dx$

$=\displaystyle\int_{x_1}^{x_2}\left(\frac{3}{4}-x-x^2\right)dx$

Now, $1+x^2=-x+\dfrac{7}{4}$

$\Rightarrow\ 4x^2+4x-3=0\ \Rightarrow\ x=-\dfrac{3}{2}$ and $x=\dfrac{1}{2}$

$\therefore\ A=\displaystyle\int_{-3/2}^{1/2}\left(\frac{3}{4}-x-x^2\right)dx=\left[\frac{3}{4}x-\frac{x^2}{2}-\frac{x^3}{3}\right]_{-3/2}^{1/2}$

$\Rightarrow\ A=\dfrac{4}{3}$ sq. units $\Rightarrow$ $3A=4$.

14. (a) : We have, $2^{n+10} = 2\times 2^2 + 3\times 2^3 + 4\times 2^4 + \dots +$
$(n-1)\times 2^{n-1} + n\times 2^n$ ...(i)

On multiplying both sides by 2, we get

$2\times 2^{n+10} = 2\times 2^3 + 3\times 2^4 + \dots + (n-1)2^n + n\times 2^{n+1}$ ...(ii)

Subtracting (ii) from (i), we get

$-2^{n+10} = 2\times 2^2 + (2^3 + 2^4 + \dots + 2^n) - n\times 2^{n+1}$

$\Rightarrow\ -2^{n+10} = 8 + 8(2^{n-2} - 1) - n\times 2^{n+1}$

$\Rightarrow\ -2^{n+10} = 2^{n+1} - n\times 2^{n+1}$

$\Rightarrow\ -2^{10} = 2 - 2n \Rightarrow n = 513$

15. (b): Since $\frac{a}{b} = \frac{2+\sqrt{3}}{1} \Rightarrow \angle A > \angle B$

Also, $\frac{a}{b} = \frac{\sin 105^\circ}{\sin 15^\circ} = 2+\sqrt{3}$, which is true.

16. (c) : Here $f(x) = x^4 + ax^3 + \frac{3x^2}{2} + 1$

$\Rightarrow f'(x) = 4x^3 + 3ax^2 + 3x \Rightarrow f''(x) = 12x^2 + 6ax + 3$

Now, $f(x)$ will be concave upward along the entire real line if $f''(x) \ge 0\ \forall\ x \in R$

$\Rightarrow\ 12x^2 + 6ax + 3 \ge 0\ \forall\ x \in R \ \Rightarrow\ D \le 0$

$\Rightarrow\ 36a^2 - 144 \le 0 \ \Rightarrow\ a^2 - 4 \le 0 \ \Rightarrow\ a \in [-2, 2]$

17. (b): $\because x^2 - x + 1 = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} > 0$

For $\sin^{-1}\sqrt{x^2 - x + 1},\ 0 < x^2 - x + 1 \le 1$

or $x^2 - x \le 0$ ...(i)

For $\cos^{-1}\sqrt{x^2 - x},\ x^2 - x \ge 0$ ...(ii)

From (i) and (ii), $x^2 - x = 0 \Rightarrow x = 0, 1$

18. (a,c) : $f(x) = [\tan^2 x] = 0$ for $-\pi/4 < x < \pi/4$.

Thus $\lim_{x\to 0} f(x)$ exists and the value is 0. Moreover, it is continuous at $x = 0$. Being a constant function f is differentiable at $x = 0$ and $f'(0) = 0$.

19. (a,b,c,d) : Given, $P(A) = 1/2$, $P(B) = 2/3$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{2}{3} - P(A \cap B)$

$\therefore\ P(A \cup B) = \frac{7}{6} - P(A \cap B) \ge \frac{2}{3} \ \Rightarrow\ P(A \cap B) \le \frac{7}{6} - \frac{2}{3}$

$\Rightarrow\ P(A \cap B) \le \frac{1}{2}$ which is true. ...(i)

Now, $P(A \cup B') = P(A) + P(B') - P(A \cap B')$

$\Rightarrow\ P(A \cap B') = P(A) + P(B') - P(A \cup B')$

$\le \frac{1}{2} + \frac{1}{3} - \frac{1}{2} = \frac{1}{3}$

Also, $P(A \cap B') = P(A) - P(A \cap B) \le \frac{1}{3}$

$\Rightarrow\ \frac{1}{2} - P(A \cap B) \le \frac{1}{3} \ \Rightarrow\ P(A \cap B) \ge \frac{1}{2} - \frac{1}{3}$

$\therefore\ P(A \cap B) \ge \frac{1}{6}$ which is true. ...(ii)

From (i) and (ii), $\frac{1}{6} \le P(A \cap B) \le \frac{1}{2}$ ...(iii)

From (iii), $-\frac{1}{6} \ge -P(A \cap B) \ge -\frac{1}{2}$

or $P(B) - \frac{1}{6} \ge P(B) - P(A \cap B) \ge P(B) - \frac{1}{2}$

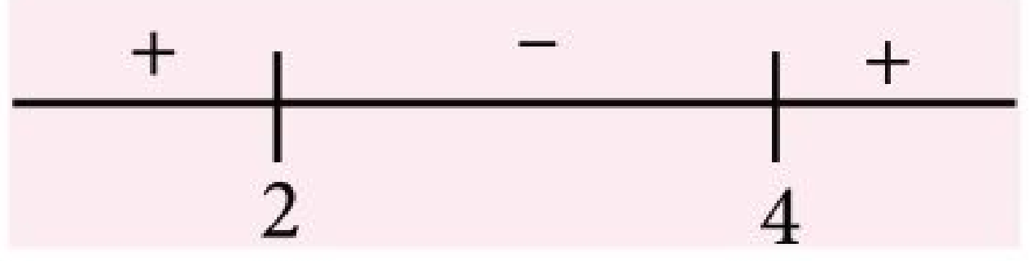
$\Rightarrow\ \frac{2}{3} - \frac{1}{6} \ge P(A' \cap B) \ge \frac{2}{3} - \frac{1}{2} \Rightarrow \frac{1}{6} \le P(A' \cap B) \le \frac{1}{2}$

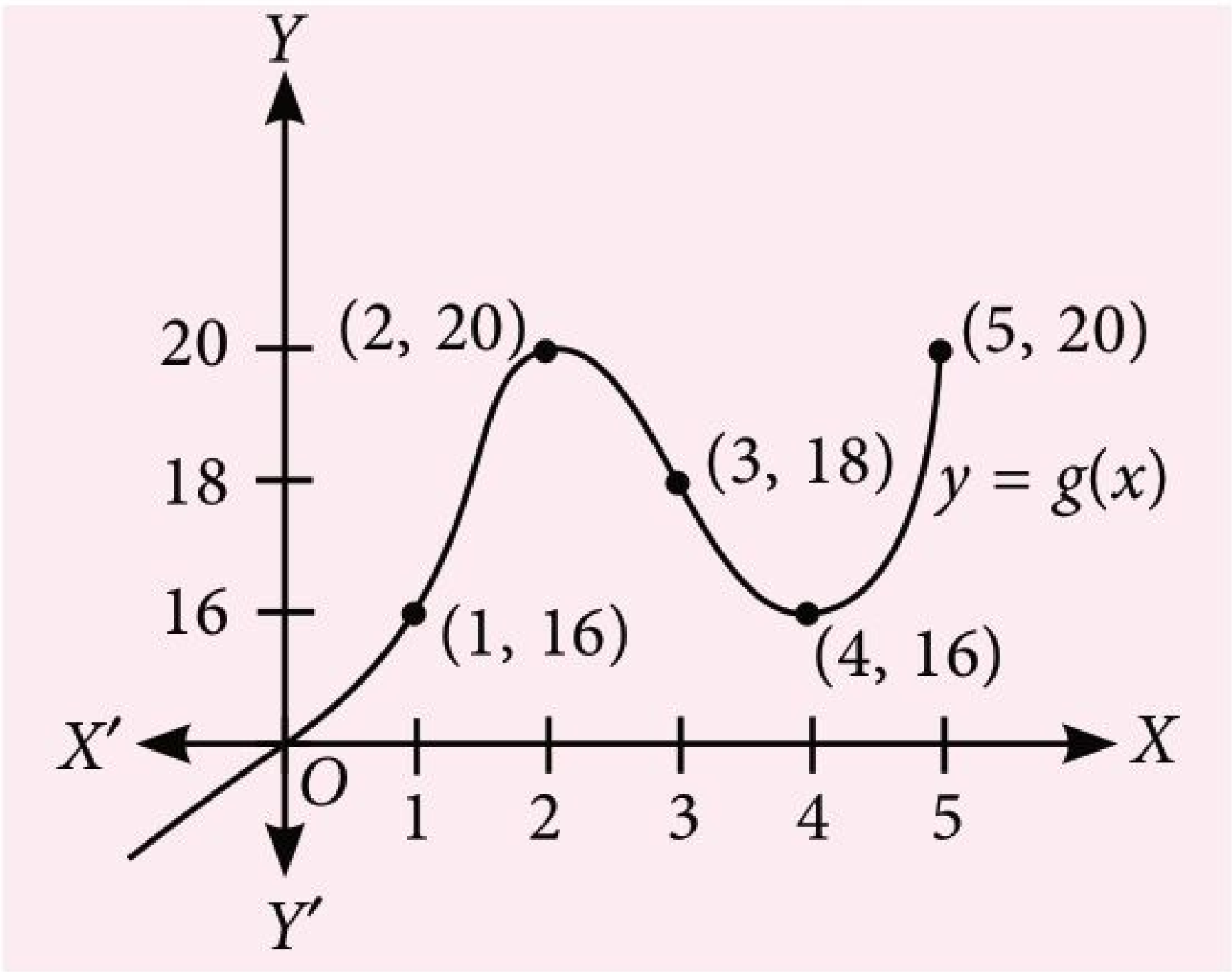
20. (b, c, d) : We have 770 = (2) (5) (7) (11). We can assign each of 2, 5, 7, 11 in 4 ways. Thus, required number of solutions is $4^4 = 256 = 16^2 = 2^8$

(21-23) : Let $g(x) = x^3 - 9x^2 + 24x = x(x^2 - 9x + 24)$

$\therefore\ g'(x) = 3(x-2)(x-4)$

Sign scheme of $g'(x)$





For three real roots of $f(x) = x^3 - 9x^2 + 24x + c = 0$, c must be lie in the interval $(-20, -16)$.

$f(0) = c < 0$

$f(1) = 1 - 9 + 24 + c = c + 16 < 0$ for $\forall c \in (-20, -16)$

$f(2) = 8 - 36 + 48 + c = c + 20 > 0$

$\alpha \in (1, 2)$ or $[\alpha] = 1$

$f(3) = 27 - 81 + 72 + c = 18 + c$

21. (a) **22. (c)** **23. (b)**

(24-25) : Deduction from the sequence of integers in the form of groups

$$(1), (2, 2), (3, 3, 3), \dots, \underbrace{(n, n, n \dots, n)}_{n\text{-times}}$$

- The number of digits in 1st, 2nd, 3rd, ..., nth group are respectively 1, 2, 3, 4, ..., n.
- Sum of the digits of 1st, 2nd, 3rd, ..., nth group are $1^2, 2^2, 3^2, \dots, n^2$. Therefore sum of the terms upto nth group is $1^2 + 2^2 + 3^2 + \dots + n^2$.
- Sum of the different terms upto nth group is $\frac{n(n+1)}{2}$, which also represent the total number of terms upto nth group.

- In each group each digit is same in single digit number and in two digits it occurs in identical pair.
- The first term of $(n + 1)^{\text{th}}$ group is $\left(\frac{n(n+1)}{2}+1\right)^{\text{th}}$ term.

24. (c) : Again total terms upto 46th group is

$$\frac{46\times 47}{2}=1081$$

∴ Sum of first 1100 terms

$$= 1^2 + 2^2 + \ldots + 46^2 + (1100 - 1081) \times 47$$

$$=\frac{46\times 47\times 93}{6}+47\times 19 = 33511 + 893 = 34404$$

25. (a) : As upto 47th group the total number of terms

$$=\frac{47\times 48}{2}=1128 \text{ (each having value 47)}$$

∴ Terms left for counting after 1100th term $= 1128 - 1100 = 28$

∴ Required sum of remaining 28 terms $= 47 \times 28 = 1316$

(26-28) : Let a be the first term and r be the common ratio of the given G.P. Further, let there be n terms in the given G.P. Then,

$$a_1 + a_n = 66 \Rightarrow a + ar^{n-1} = 66 \quad \ldots\text{(i)}$$

$$a_2 \times a_{n-1} = 128 \Rightarrow ar \times ar^{n-2} = 128 \Rightarrow a^2 r^{n-1} = 128$$

$$\Rightarrow a\times(ar^{n-1})=128 \Rightarrow ar^{n-1}=\frac{128}{a}$$

Putting this value of ar^{n-1} in (i), we get

$$a+\frac{128}{a}=66 \Rightarrow a^2 - 66a + 128 = 0$$

$$\Rightarrow (a - 2)(a - 64) = 0 \Rightarrow a = 2, 64$$

Putting $a = 2$ in (i), we get $2 + 2 \times r^{n-1} = 66 \Rightarrow r^{n-1} = 32$

Putting $a = 64$ in (i), we get $64+64r^{n-1}=66 \Rightarrow r^{n-1}=\frac{1}{32}$

For an increasing G.P., $r > 1$.

Now, $S_n = 126$

$$\Rightarrow 2\left(\frac{r^n-1}{r-1}\right)=126 \Rightarrow \frac{r^n-1}{r-1}=63 \Rightarrow \frac{r^{n-1}\times r-1}{r-1}=63$$

$$\Rightarrow \frac{32r-1}{r-1}=63 \quad \Rightarrow \quad r=2$$

∴ $r^{n-1} = 32 \Rightarrow 2^{n-1} = 32 = 2^5 \Rightarrow n - 1 = 5 \Rightarrow n = 6$

For decreasing G.P., $a = 64$ and $r = 1/2$. Hence, the sum of infinite terms is $64/[1 - (1/2)] = 128$.

For $a = 2$, $r = 2$, terms are 2, 4, 8, 16, 32, 64.

For $a = 64$, $r = \frac{1}{2}$, terms are 64, 32, 16, 8, 4, 2.

∴ Difference = 62.

26. (d) **27. (b)** **28. (d)**

29. (A) → (S); (B) → (R); (C) → (P); (D) → (Q)

(A) $L=\lim_{x\to-1}\frac{\sqrt[3]{7-x}-2}{x+1}=\lim_{h\to0}\frac{(8-h)^{1/3}-2}{h}$

(put $x + 1 = h$, as $x \to -1 \Rightarrow h \to 0$)

$$=\lim_{h\to0}\frac{2\left[1-\frac{h}{8}\right]^{\frac{1}{3}}-2}{h}=2\left[\lim_{h\to0}\frac{\frac{1}{3}\left(1-\frac{h}{8}\right)^{-2/3}\left(-\frac{1}{8}\right)}{1}\right]$$

$$=2\left(-\frac{1}{24}\right)=\frac{-1}{12} \Rightarrow \quad 12L = -1$$

(B) $L=\lim_{x\to\frac{\pi}{4}}\left[\frac{\tan^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}\right]$

$$=\lim_{x\to\frac{\pi}{4}}\left[\tan x\frac{(\tan x-1)(\tan x+1)}{\cos\left(x+\frac{\pi}{4}\right)}\right]$$

$$=\lim_{x\to\frac{\pi}{4}}\left[\frac{\tan x(\sin x-\cos x)(\tan x+1)}{\cos x\cos\left(x+\frac{\pi}{4}\right)}\right]$$

$$=-\sqrt{2}\lim_{x\to\frac{\pi}{4}}\frac{\tan x\left[\cos x\frac{1}{\sqrt{2}}-\sin x\frac{1}{\sqrt{2}}\right](\tan x+1)}{\cos x\cos\left(x+\frac{\pi}{4}\right)}$$

$$=-\sqrt{2}\lim_{x\to\frac{\pi}{4}}\frac{\tan x\cos\left(x+\frac{\pi}{4}\right)(\tan x+1)}{\cos x\cos\left(x+\frac{\pi}{4}\right)}$$

$$=-\sqrt{2}\times2\times\sqrt{2}=-4 \Rightarrow -(L/4)=1$$

(C) $L=\lim_{x\to1}\frac{(2x-3)(\sqrt{x}-1)}{(2x+3)(\sqrt{x}+1)(\sqrt{x}-1)}=\frac{-1}{10}$

∴ $20L = -2$

(D) $L=\lim_{x\to\infty}\frac{\log x^n-[x]}{[x]}=\lim_{x\to\infty}\frac{\log x^n}{[x]}-\lim_{x\to\infty}\frac{[x]}{[x]}$

$= 0 - 1 = -1$ ∴ $-2L = 2$

30. (A) → (R); (B) → (P); (C) → (Q); (D) → (S)

(A) $\overrightarrow{OA}=\hat{i}+2\hat{j}+3\hat{k},\ \overrightarrow{OB}=-2\hat{i}+\hat{j}-4\hat{k},$

$\overrightarrow{OC}=3\hat{i}+4\hat{j}-2\hat{k}$

∴ Required area $=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|=\frac{\sqrt{1218}}{2}$

(B) $((\vec{a}\times\vec{b})\times\vec{c})\cdot\vec{d}+((\vec{b}\times\vec{c})\times\vec{a})\cdot\vec{d}+((\vec{c}\times\vec{a})\times\vec{b})\cdot\vec{d}=0$

(C) Since, the given lines intersect.

$\therefore\ (2r+1, 3r-1, 4r+1) = (s+3, 2s+k, s)$

$\Rightarrow\ r = \frac{-3}{2}, s = -5 \Rightarrow k = \frac{9}{2}$

(D) Here, $\vec{a} \times \vec{b} = -10\hat{i} + 9\hat{j} + 7\hat{k}$

and $\vec{c} \times \vec{d} = -\hat{i} + 2\hat{j} - \hat{k}$

$\therefore\ (\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 10 + 18 - 7 = 21$

31. **(A) → (U); (B) → (R); (C) → (S, T); (D) → (P, Q)**

(A) $\cot 82\frac{1}{2}^{\circ} = \tan 7\frac{1}{2}^{\circ} = \dfrac{\sin 7\frac{1}{2}^{\circ}}{\cos 7\frac{1}{2}^{\circ}}$

$$= \frac{2\sin^2 7\frac{1}{2}^{\circ}}{2\sin 7\frac{1}{2}^{\circ}\ \cos 7\frac{1}{2}^{\circ}} = \frac{1-\cos 15^{\circ}}{\sin 15^{\circ}} = \frac{1-\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$= \frac{(2\sqrt{2}-\sqrt{3}-1)(\sqrt{3}+1)}{2}$$

$$= \sqrt{6}-\sqrt{3}+\sqrt{2}-2 = (\sqrt{3}-\sqrt{2})(\sqrt{2}-1)$$

(B) $\cot 15^{\circ} = \dfrac{\cos 15^{\circ}}{\sin 15^{\circ}} = \dfrac{2\cos^2 15^{\circ}}{2\sin 15^{\circ}\cos 15^{\circ}}$

$$= \frac{1+\cos 30^{\circ}}{\sin 30^{\circ}} = 2+\sqrt{3}$$

(C) $\tan 82\frac{1}{2}^{\circ} = \cot 7\frac{1}{2}^{\circ} = \dfrac{\cos 7\frac{1}{2}^{\circ}}{\sin 7\frac{1}{2}^{\circ}} = \dfrac{2\cos^2 7\frac{1}{2}^{\circ}}{2\sin 7\frac{1}{2}^{\circ}\ \cos 7\frac{1}{2}^{\circ}}$

$$= \frac{1+\cos 15^{\circ}}{\sin 15^{\circ}} = \frac{1+\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{2\sqrt{2}+\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \sqrt{6}+\sqrt{3}+\sqrt{2}+2 = \left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{2}+1\right)$$

(D) $\tan 67\frac{1}{2}^{\circ} = \cot 22\frac{1}{2}^{\circ} = \dfrac{\cos 22\frac{1}{2}^{\circ}}{\sin 22\frac{1}{2}^{\circ}}$

$$= \frac{2\cos^2 22\frac{1}{2}^{\circ}}{2\sin 22\frac{1}{2}^{\circ}\ \cos 22\frac{1}{2}^{\circ}} = \frac{1+\cos 45^{\circ}}{\sin 45^{\circ}} = \sqrt{2}+1$$

32. (1980) : Required sum

$= (12 + 14 + 16 + \ldots + 98) - (20 + 30 + 40 + \ldots + 90)$

$= \frac{44}{2}(12+98) - \frac{8}{2}(20+90)$

$= 22(110) - 4(110)$

$= (110)(18) = 1980$

33. (1762) : Here, $\alpha = 2\sin^{-1}\left(\frac{2}{3}\right)$

$$= \sin^{-1}\left[2 \times \frac{2}{3}\sqrt{1-\frac{4}{9}}\right] = \sin^{-1}\frac{4\sqrt{5}}{9}$$

$\Rightarrow\ 81\sin^2\alpha = 80 \Rightarrow 80\,\text{cosec}^2\alpha = 81$

Also, $\beta = 2\tan^{-1} 9 = \sin^{-1}\dfrac{2 \times 9}{1+81} = \sin^{-1}\dfrac{9}{41}$

$\Rightarrow\ \sin\beta = \frac{9}{41}\ \Rightarrow 81\,\text{cosec}^2\beta = 1681$

$\therefore\ 80\text{cosec}^2\alpha + 81\text{cosec}^2\beta = 1681 + 81 = 1762$

34. (2): Since 1 + 2 + ... + 9 = 45, a number consisting all these digits will be divisible by 9. Thus, the number will be divisible by 36 if and only if it is divisible by 4. There are ${}^9P_9 = 9!$ ways of arranging the nine digits to form a nine-digit number. If this number is to be divisible by 4, its last digit must be even, and the number formed by its last two digits must be divisible by 4.

∴ Possible values of the last pair must be as : 12, 32, 52, 72, 92, 24, 64, 84, 16, 36, 56, 76, 96, 28, 48, 68.

That is, there are 16 ways of choosing the last two digits. The remaining digits can be arranged in ${}^7P_7 = 7!$ ways. Therefore, the number of favourable arrangements is (7!) (16) out of a total of 9!

$\therefore$ Required probability $= \dfrac{16(7!)}{9!} = \dfrac{16}{9 \times 8} = \dfrac{2}{9}$

35. (164): The probability of getting a double-six in one throw with two dice $= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

$\therefore\ p = \frac{1}{36}, q = 1 - p = 1 - \frac{1}{36} = \frac{35}{36}$

Now, $(p+q)^n = p^n + {}^nC_1\, p^{n-1}q + {}^nC_2\, p^{n-2}q^2 + \ldots + {}^nC_{n-1}pq^{n-1} + q^n$

The probability of getting atleast one double-six in n throws with two dice $= (p+q)^n - q^n = 1 - q^n$

$$= 1 - \left(\frac{35}{36}\right)^n \quad \therefore\ 1 - \left(\frac{35}{36}\right)^n > 0.99$$

$\Rightarrow\ \left(\frac{35}{36}\right)^n < 0.01 \Rightarrow n[\log_{10} 35 - \log_{10} 36] < \log_{10} 0.01$

$\Rightarrow\ n[1.5441 - 1.5563] < -2.0000 \Rightarrow -0.0122n < -2$

$\Rightarrow\ 0.0122n > 2\ \Rightarrow\ n > \dfrac{2}{0.0122} \Rightarrow n > 163.9$

$\therefore$ The least value of n is 164.

36. (1): $y = \dfrac{1}{1-x}$

At $x = 2, y = -1$

Now, $\frac{dy}{dx}=\frac{1}{(1-x)^2}$ ∴ $\left[\frac{dy}{dx}\right]_{(2,-1)}=1$

So, equation of tangent to the curve is

$$y+1=1(x-2) \quad \text{or} \quad y=x-3$$

Solving this equation of line with $y=-a^2x^2+5ax-4$, we get

$$x-3=-a^2x^2+5ax-4$$

or $\quad a^2x^2+x(1-5a)+1=0$ ...(i)

Above equation has two roots x_1 and x_2 which are abscissas of point of intersection P and Q.

Mid point of PQ lies on $x=2$

$$\therefore \quad \frac{x_1+x_2}{2}=2 \quad \Rightarrow \quad x_1+x_2=4$$

From equation (i), $x_1+x_2=-\left(\frac{1-5a}{a^2}\right) \Rightarrow 4=\frac{5a-1}{a^2}$

$$\Rightarrow \quad 4a^2-5a+1=0 \quad \Rightarrow \quad a=1/4,\ 1$$

But for $a=1/4$, eq. (i) gives imaginary roots.

37. (3): $\tan\theta=\cot 5\theta=\tan\left(\frac{\pi}{2}-5\theta\right)$

$$\Rightarrow \theta=n\pi+\frac{\pi}{2}-5\theta \Rightarrow \theta=(2n+1)\frac{\pi}{12}$$

$$\therefore \ \theta=\frac{\pi}{12},\frac{\pi}{4},\frac{5\pi}{12},\frac{-\pi}{12},\frac{-\pi}{4},\frac{-5\pi}{12} \qquad \text{...(i)}$$

Now, $\cos 4\theta=\sin 2\theta=\cos\left(\frac{\pi}{2}-2\theta\right)$

$$\therefore \quad 4\theta=2n\pi\pm\left(\frac{\pi}{2}-2\theta\right)$$

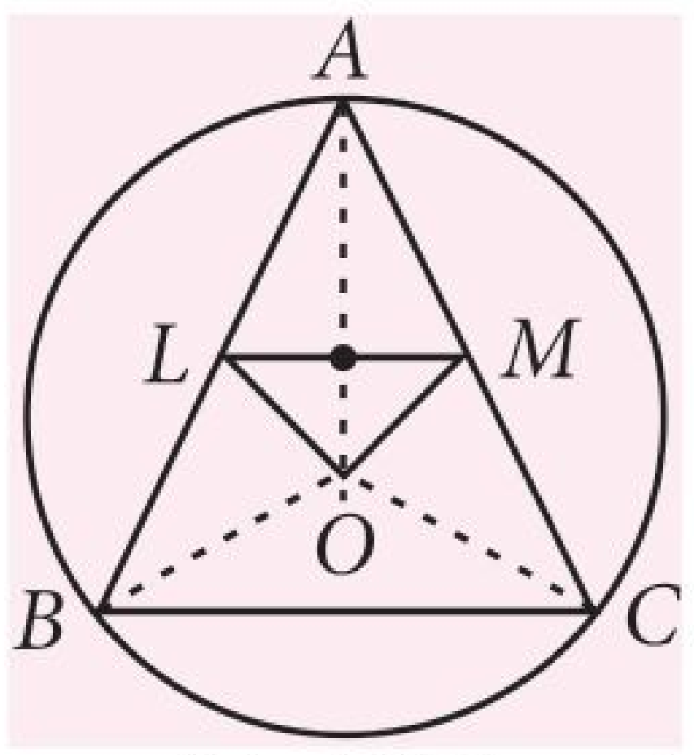
$$\Rightarrow \quad \theta=(4n+1)\frac{\pi}{12},\ (4n-1)\frac{\pi}{4} \qquad \text{...(ii)}$$

$$\theta=\frac{\pi}{12},\frac{5\pi}{12},\frac{-\pi}{4}$$

The common values of (i) and (ii) are in (ii). The number of values of θ is 3.

38. (4) : Let $AB=c$, $BC=a$ and $AC=b$.

As L, M are midpoints of AB and AC respectively.



$$\therefore \quad LM=\frac{1}{2}BC=\frac{a}{2}$$

Also, $OA=OB=OC=R=$ circumradius of ΔABC and $\angle AOM=\angle B$ and $OM=OA\cos B=R\cos B$, now $\angle AML=\angle C$

Since, $LM \parallel BC$ as L, M are midpoints of AB and AC respectively.

$$\therefore \quad \angle LMO=\frac{\pi}{2}-\angle C$$

$$\therefore \quad \text{Area of triangle } OLM=\frac{1}{2}LM\cdot OM\sin\angle LMO$$

$$=\frac{1}{2}\frac{BC}{2}(R\cos B)\cos C=\frac{aR\cos B\cos C}{4}$$

We know that,

$$\text{area of } \Delta ABC=\frac{1}{2}ab\sin C=\frac{a}{2}(2R\sin B\sin C)$$

$=aR\sin B\sin C$

$$\text{Thus, } \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta OLM)}=\frac{aR\sin B\sin C}{\frac{aR}{4}\cos B\cos C}=4\tan B\tan C$$

$$\Rightarrow \quad \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta OLM)}\cot B\cot C=4$$

39. (2): From the given equation

$\tan^2x+\cot^2x+1+\cot^2x+4\sin^2x=6$

$$\Rightarrow \quad \tan^2x+2\cot^2x+\frac{4\sin^2x}{\sin^2x+\cos^2x}=5$$

$$\Rightarrow \quad \tan^2x+2\cot^2x+\frac{4\tan^2x}{1+\tan^2x}=5$$

$$\Rightarrow \quad t^2+\frac{2}{t^2}+\frac{4t^2}{1+t^2}=5 \qquad \text{(By putting } \tan x=t\text{)}$$

$$\Rightarrow \quad t^6-3t^2+2=0$$

$$\Rightarrow \quad (t^2-1)(t^4+t^2-2)=0 \Rightarrow (t^2-1)^2(t^2+2)=0$$

$$\Rightarrow \quad t^2=1 \Rightarrow \tan x=\pm 1$$

$\Rightarrow \quad x=\frac{\pi}{4},\frac{3\pi}{4}$ as $x\in[0,\pi]$ only two values are possible

and $t^4+t^2-2=0$ rejected as either values are repeated in $[0,\pi]$ or does not belongs to $[0,\pi]$.

∴ Only two values are possible in $[0,\pi]$.

40. (8): $\because \quad \cos(A-B)=\frac{1-\tan^2\left(\frac{A-B}{2}\right)}{1+\tan^2\left(\frac{A-B}{2}\right)}$

$$\Rightarrow \quad \frac{251}{256}=\frac{1-\tan^2\left(\frac{A-B}{2}\right)}{1+\tan^2\left(\frac{A-B}{2}\right)}$$

$$\Rightarrow \quad \tan\left(\frac{A-B}{2}\right)=\sqrt{\frac{5}{507}} \Rightarrow \frac{a-b}{a+b}\cot\frac{C}{2}=\sqrt{\frac{5}{507}}$$

$$\Rightarrow \cot\frac{C}{2}=\frac{13\sqrt{5}}{\sqrt{507}} \quad \Rightarrow \quad \tan\frac{C}{2}=\frac{\sqrt{507}}{13\sqrt{5}}$$

$$\text{Again, } \cos C=\frac{1-\tan^2 C/2}{1+\tan^2 C/2}=\frac{1-\frac{507}{845}}{1+\frac{507}{845}}=\frac{338}{1352}=\frac{1}{4}$$

$$\Rightarrow \quad \frac{a^2+b^2-c^2}{2ab}=\frac{1}{4} \quad \Rightarrow \quad \frac{49+36-c^2}{2\cdot 7\cdot 6}=\frac{1}{4}$$

$$\Rightarrow \quad c^2=64 \quad \Rightarrow \quad c=8$$

◆◆

beat the TIME TRAP

Duration : 30 minutes

SECTION-I

Single Option Correct Type

1. The cubes of the natural numbers are grouped as $1^3, (2^3, 3^3), (4^3, 5^3, 6^3)$ ...., then sum of the numbers in the n^{th} group is

(a) $\frac{n^2}{12}(n^2+1)(n^2+4)$

(b) $\frac{n^3}{8}(n^2+1)(n^2+3)$

(c) $\frac{n^3}{8}(n^2+2)(n^2+4)$

(d) $\frac{n^2}{16}(n^2+1)(n^2+4)$

2. The position vector of a point P is $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, where $x, y, z \in N$ and $\vec{a} = \hat{i} + \hat{j} + \hat{k}$. If $\vec{r}\cdot\vec{a} = 12$, then number of possible position of P is

(a) 45 (b) 55
(c) 35 (d) None of these

3. Area bounded by the curve $x^2 + y^2 \le 8x$ and $y^2 \ge 4x$ lies in the first quadrant is not equal to

(a) $32\left(\frac{3\pi}{8} - 1\right)$ sq. units

(b) $\frac{32}{3}\left(\frac{3\pi}{8} - 1\right)$ sq. units

(c) $\left(4\pi - \frac{32}{3}\right)$ sq. units

(d) $\frac{1}{3}(12\pi - 32)$ sq. units

4. If p^{th}, q^{th}, r^{th} terms of a G.P. are the positive numbers a, b, c, then angle between the vectors

$\log a^3\hat{i} + \log b^3\hat{j} + \log c^3\hat{k}$ and

$(q-r)\hat{i} + (r-p)\hat{j} + (p-q)\hat{k}$, is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{a^2+b^2+c^2}}\right)$

5. If A is a square matrix, B is singular matrix of same order, then for a positive integer n, $(A^{-1}BA)^n$ equals to

(a) $A^{-n}B^nA^n$ (b) $A^nB^nA^{-n}$
(c) $A^{-1}B^nA$ (d) $n(A^{-1}BA)$

6. For three events A, B and C, P(Exactly one of A or B occurs) $= P$(Exactly one of B or C occurs) $= P$(Exactly one of C or A occurs) $= \frac{1}{4}$ and P(All the three events occur simultaneously) $= \frac{1}{16}$. Then the probability that at least one of the events occurs, is

(a) $\frac{7}{16}$ (b) $\frac{7}{64}$

(c) $\frac{3}{16}$ (d) $\frac{7}{32}$

7. Consider the following two statements :

P : If 7 is an odd number, then 7 is divisible by 2.

Q : If 7 is a prime number, then 7 is an odd number.

If V_1 is the truth value of the contrapositive of P and V_2 is the truth value of contrapositive of Q, then the ordered pair (V_1, V_2) equals

(a) (F, F) (b) (F, T)
(c) (T, F) (d) (T, T)

8. The number of roots of the equation

$\cos 3x + \cos 2x = \sin\frac{3x}{2} + \sin\frac{x}{2}$ in $[0, 2\pi]$ is

(a) 2 (b) 3
(c) 4 (d) 5

9. If $\lim\limits_{x\to 0}\dfrac{a\sin x-bx+cx^2+x^3}{2x^2\ln(1+x)-2x^3+x^4}=L$, where L is finite and non-zero, then $L=$

(a) $\dfrac{1}{20}$ (b) $\dfrac{3}{20}$ (c) $\dfrac{3}{40}$ (d) $\dfrac{5}{41}$

10. If $y=e^{2\sin^{-1}x}$, then

(a) $(1+x^2)y''-xy'+4y=0$
(b) $(1+x^2)y''+xy'-4y=0$
(c) $(1-x^2)\,y''+xy'-2y=0$
(d) $(1-x^2)\,y''-xy'-4y=0$

SECTION-II

Numerical Answer Type

11. If the normal at the end points of a variable chord AB of the parabola $y^2-8y+2x=0$ are perpendicular then the tangents at A and B will intersect at $ax+b=0$ then value of $|a+b|$ equals ________.

12. If the quadratic equations $ax^2+2cx+b=0$ and $ax^2+2bx+c=0$, $(b\neq c)$ have a common root, then $(a+4b+4c)$ is ________.

13. If $I=\displaystyle\int_0^{\pi/2}\frac{x}{\sin x}\,dx$ and $J=\displaystyle\int_0^{1}\frac{\tan^{-1}x}{x}\,dx$, then $\dfrac{I}{J}$ is ________.

14. The order of differential equation whose general solution is given by

$y=C_1\cos(x+C_2)-C_3e^{C_4-x}+C_5\sin x$, where C_1, C_2, C_3, C_4, C_5 are arbitrary constants is ________.

15. If the foot of perpendicular from the point $P(4, 6, 2)$ to the line $x-y-z+2=0=2x-y-4z+6$ is (a, b, c), then $\dfrac{a+b+c}{3}$ is ________.

SOLUTIONS

1. (b) : Series using the last element of the each group is $1^3, 3^3, 6^3, 10^3, \ldots$

$\therefore$ t_n of 1, 3, 6, 10, ... is $\dfrac{1}{2}n(n+1)$

Now, if we replace n by $n-1$, we get the last element of $(n-1)^{\text{th}}$ group

$\therefore$ Last element of the $(n-1)^{\text{th}}$ group $=\dfrac{1}{2}n(n-1)$, if we add 1 in it and cubing the number that number be the first element of n^{th} group. So the terms in the n^{th} group are

$$\left(\frac{n(n-1)}{2}+1\right)^3, \left(\frac{n(n-1)}{2}+2\right)^3, \ldots \left(\frac{n(n+1)}{2}\right)^3$$

$\therefore$ Sum of the elements of n^{th} group

$=$ sum of the cubes of first $\dfrac{n(n+1)}{2}$ natural numbers

$-$ sum of the cubes of first $\dfrac{n(n-1)}{2}$ natural numbers

$$=\frac{\left(\frac{n(n+1)}{2}\right)^2\left(\frac{n(n+1)}{2}+1\right)^2}{4}-\frac{\left(\frac{n(n-1)}{2}\right)^2\left(\frac{n(n-1)}{2}+1\right)^2}{4}$$

$=\dfrac{n^3}{8}(n^2+1)(n^2+3)$ (On simplification)

2. (b) : Given, $\vec{r}\cdot\vec{a}=12$

$\Rightarrow$ $x+y+z=12$ and $x, y, z\in N$

$\therefore$ Required number of positions of P

= The number of positive solutions of the equation $x+y+z=12$

= The coefficient of x^{12} in $(x+x^2+x^3+\ldots)^3$

= Coefficient of x^{12} in $x^3(1+x+x^2+\ldots)^3$

= Coefficient of x^9 in $((1-x)^{-1})^3$

= Coefficient of x^9 in $(1-x)^{-3}$

$=(1+{}^3C_1x+{}^4C_2x^2+\ldots.+{}^{11}C_9x^9+\ldots.)$

$={}^{9+3-1}C_{3-1}={}^{11}C_2=\dfrac{11}{2}\cdot\dfrac{10}{1}=55$

3. (a) : Given, $x^2+y^2\le 8x$ and $y^2\ge 4x$

Let $x^2+y^2=8x$ ...(i)

and $y^2=4x$ ...(ii)

$\therefore$ From (i) and (ii), we have

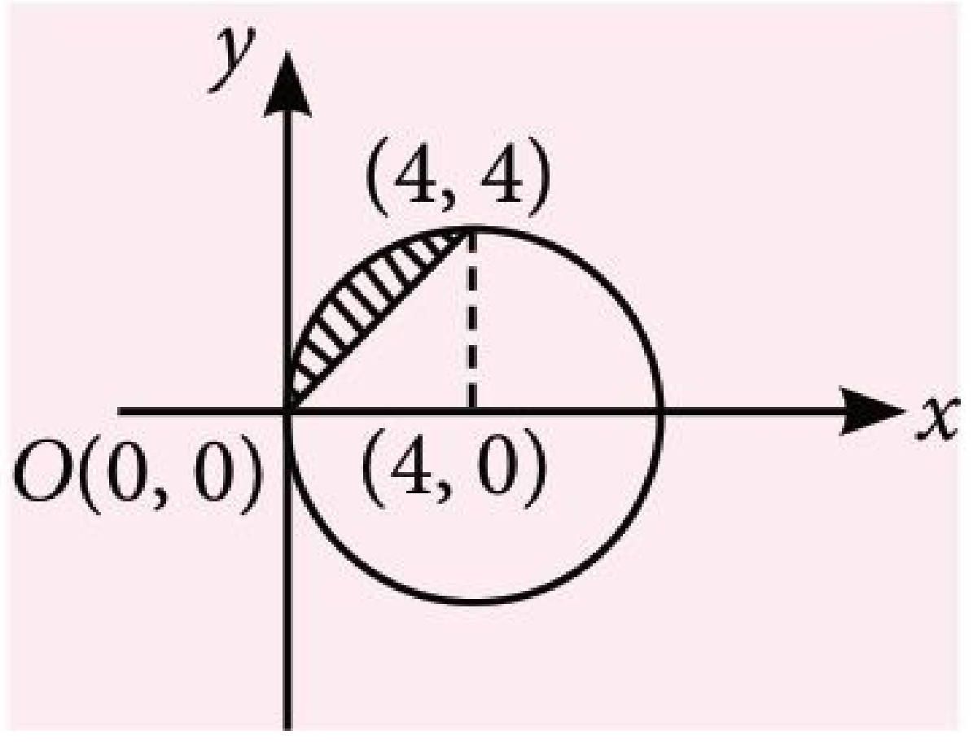
$x^2=4x$ $\therefore$ $x=0, x=4$

$\therefore$ Set of points, (or points of intersection) are (0, 0), (4, 4).

$\therefore$ Required area

$$=\int_0^4\sqrt{4^2-(x-4)^2}\,dx-\int_0^4 2\sqrt{x}\,dx$$

$$\left[\begin{aligned}&\because x^2+y^2=8x\\&\Rightarrow(x-4)^2+y^2=4^2\\&\Rightarrow y^2=4^2-(x-4)^2\end{aligned}\right]$$



$$=\left[\frac{(x-4)}{2}\sqrt{4^2-(x-4)^2}+\frac{16}{2}\sin^{-1}\left(\frac{x-4}{4}\right)\right]_0^4$$

$$-2\times\frac{2}{3}(x^{3/2})_0^4$$

$$=8\times\frac{\pi}{2}-\frac{4}{3}\times(4)^{3/2}=\left(4\pi-\frac{32}{3}\right)=\frac{32}{3}\left(\frac{3\pi}{8}-1\right)\text{ sq. unit}$$

4. (b) : Let x_0 be the first term and x be the common ratio.

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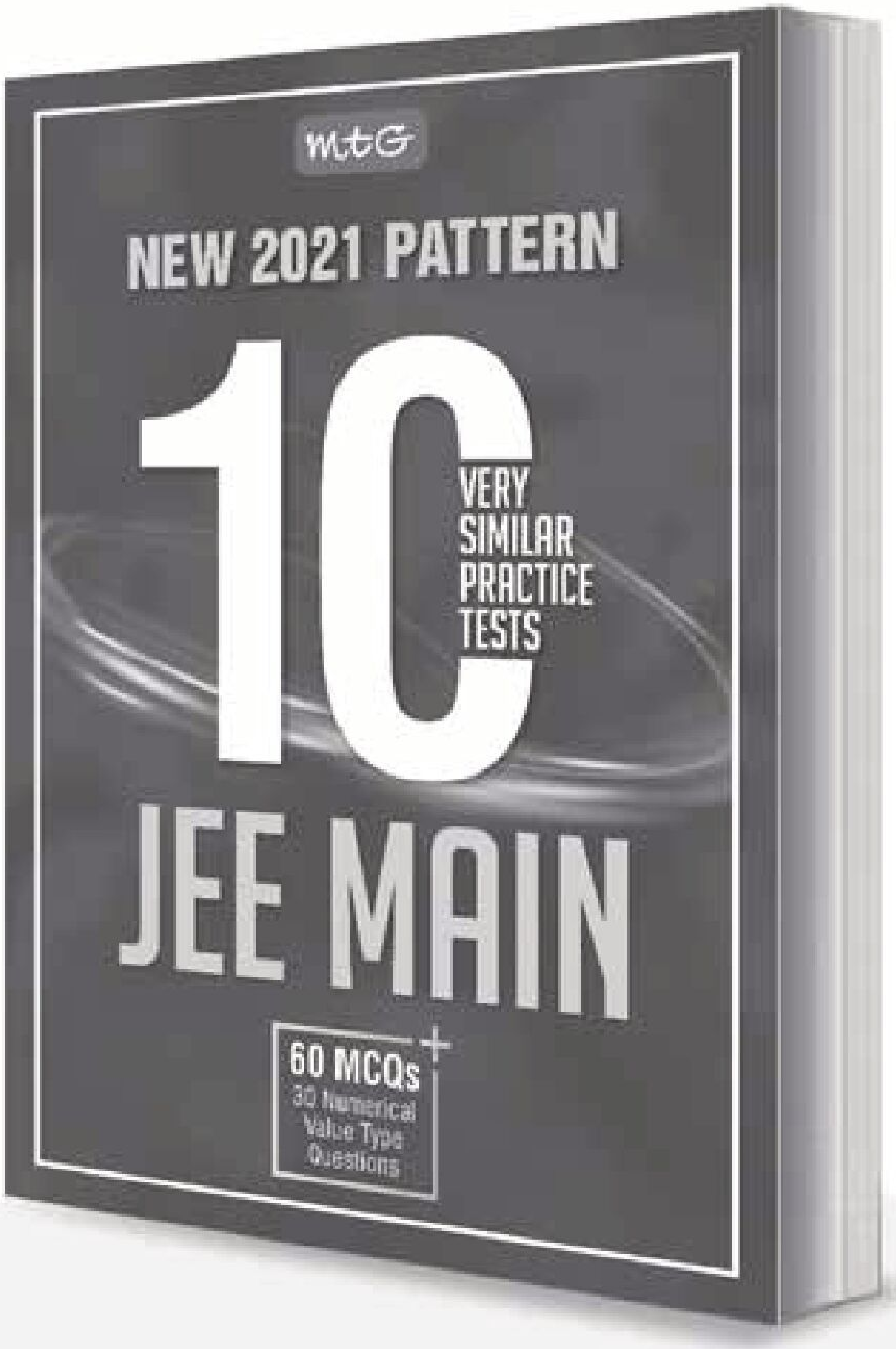

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$\therefore\ a = x_0\, x^{p-1}, b = x_0\, x^{q-1}, c = x_0\, x^{r-1}$

$\therefore\ \log a = \log x_0 + (p-1)\log x$ etc.

Let $\vec{a} = \log a^3\hat{i} + \log b^3\hat{j} + \log c^3\hat{k}$

$\vec{b} = (q-r)\hat{i} + (r-p)\hat{j} + (p-q)\hat{k}$

$\therefore\ \vec{a}\cdot\vec{b} = \Sigma\, 3(\log a)(q-r)$

$= 3\sum\{\log x_0 + (p-1)\log x\}(q-r) = 0$

$\therefore\ \theta = \pi/2$

5. (c) : Consider $n = 2$

$\therefore\ (A^{-1}BA)^2 = (A^{-1}BA)(A^{-1}BA)$

$= (A^{-1}B)(AA^{-1})(BA) = A^{-1}B^2A$

$\therefore$ Again for $n = 3$ we have

$(A^{-1}BA)^3 = (A^{-1}B^2A)(A^{-1}BA) = A^{-1}B^3A$

$\therefore$ Generalizing the case

$(A^{-1}BA)^n = A^{-1}B^nA$

6. (a) : Given $P(\text{exactly one of } A \text{ or } B \text{ occurs}) = \dfrac{1}{4}$

Then, $P(A) + P(B) - 2P(A\cap B) = \dfrac{1}{4}$

Similarly, $P(B) + P(C) - 2P(B\cap C) = \dfrac{1}{4}$

Also, $P(C) + P(A) - 2P(C\cap A) = \dfrac{1}{4}$

Adding all of them, we have

$$P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) = \frac{3}{8}$$

Now, $P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$

$$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$$

7. (b) : We have,

Contrapositive of P : If 7 is not divisible by 2, then 7 is not an odd number. $T \Rightarrow F : F(V_1)$

Contrapositive of Q : If 7 is not an odd number, then 7 is not a prime number. $F \Rightarrow F : T\,(V_2)$

8. (d) : Given, $\cos 3x + \cos 2x = \sin\dfrac{3x}{2} + \sin\dfrac{x}{2}$

$$\Rightarrow\ 2\cos\frac{5x}{2}\cos\frac{x}{2} = 2\sin x\cos\frac{x}{2}$$

$$\Rightarrow \cos\frac{x}{2}\left(\cos\frac{5x}{2} - \sin x\right) = 0$$

$$\Rightarrow\ \cos\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = (2n+1)\frac{\pi}{2}$$

$\Rightarrow\ x = (2n+1)\pi\ \Rightarrow\ x = \pi$ in $[0, 2\pi]$

And, $\cos\dfrac{5x}{2} = \sin x = \cos\left(\dfrac{\pi}{2} - x\right)$

$$\Rightarrow\ \frac{5x}{2} = 2n\pi \pm \left(\frac{\pi}{2} - x\right)$$

$$\Rightarrow\ \frac{5x}{2} = 2n\pi + \frac{\pi}{2} - x \Rightarrow \frac{7x}{2} = (4n+1)\frac{\pi}{2}$$

$$\Rightarrow x = (4n+1)\frac{\pi}{7} \Rightarrow x = \frac{\pi}{7}, \frac{5\pi}{7}, \frac{9\pi}{7}, \frac{13\pi}{7}$$

or $\dfrac{5x}{2} = 2n\pi - \dfrac{\pi}{2} + x \Rightarrow \dfrac{3x}{2} = (4n-1)\dfrac{\pi}{2}$

$$\Rightarrow\ x = (4n-1)\frac{\pi}{3} \Rightarrow x = \pi$$

$\therefore$ The roots are $\dfrac{\pi}{7}, \dfrac{5\pi}{7}, \pi, \dfrac{9\pi}{7}, \dfrac{13\pi}{7}$.

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique solution.

	6	9		2	3										5	9		4	7	
8					7										6					3
2					6										7					8
					5	4	2	9				7	8	9	1					
4				3				1				4				3				9
5	2	1	9														9	2	8	5
			8					6				2					5			
			3					7				8					7			
			6	7		1	8		6		9		7	3		6	1			
								2		5		6								
									8		6									
								3		7		9								
			5	4		7	2		4		3		6	5		3	7			
			1					8				3					9			
			7					1				4					2			
1	7	4	9														1	8	5	6
2				1				4				2				6				7
					2	8	1	7				6	5	8	7					
8					4										3					2
7					3										9					3
	5	6		9	1										2	8		7	4	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

9. (c) : By series, $\lim_{x\to 0}\dfrac{a\left(x-\dfrac{x^3}{6}+\dfrac{x^5}{120}-...\right)-bx+cx^2+x^3}{2x^2\left(x-\dfrac{x^2}{2}+\dfrac{x^3}{3}-...\right)-2x^3+x^4}$

$$= \lim_{x\to 0}\frac{(a-b)x+cx^2+\left(1-\dfrac{a}{6}\right)x^3+\dfrac{a}{120}x^5+...}{\dfrac{2}{3}x^5+...}=L$$

$\Rightarrow\ a = b, c = 0, 1 - \dfrac{a}{6} = 0 \Rightarrow a = 6$

$\therefore\ L = \dfrac{a}{120}\times\dfrac{3}{2} = \dfrac{3}{40}$.

10. (d) : $y' = \dfrac{2e^{2\sin^{-1}x}}{\sqrt{1-x^2}} = \dfrac{2y}{\sqrt{1-x^2}}$

$\Rightarrow\ (1 - x^2)(y')^2 = 4y^2$

Differentiating, $(1 - x^2)\cdot 2y'\,y'' - 2x(y')^2 = 8yy'$

$\Rightarrow\ (1 - x^2)y'' - xy' - 4y = 0$.

11. (15) : As the normals are ⊥ therefore tangents will also be perpendicular & the ⊥ tangents always intersect each other at directrix. Now, equation of parabola is

$y^2 - 8y + 2x = 0$

$\Rightarrow\ (y - 4)^2 = -2x + 16$

$\Rightarrow\ (y - 4)^2 = -2(x - 8)$

$Y^2 = 4\left(-\dfrac{1}{2}\right)X$, where $X = x - 8, Y = y - 4$

$\therefore$ Equation of directrix is

$X = \dfrac{1}{2}$ i.e., $x - 8 = \dfrac{1}{2}$ or $2x - 17 = 0$

Now $ax + b = 2x - 17$

$\therefore\ |a + b| = 15$

12. (0) : $ax^2 + 2cx + b = 0$ ...(i)

$ax^2 + 2bx + c = 0$ ...(ii)

(i) and (ii) imply that

$$\frac{x^2}{2(c^2-b^2)}=\frac{x}{a(b-c)}=\frac{1}{2a(b-c)}$$

$\Rightarrow\ 4a(c^2 - b^2)(b - c) = a^2(b - c)^2$

$\Rightarrow\ -4(b + c) = a \Rightarrow a + 4b + 4c = 0$.

13. (2) : Put $x = 2t \Rightarrow dx = 2dt$

$\therefore\ I = 2\displaystyle\int_0^{\pi/4}\frac{t}{\sin t\cos t}dt = 2\int_0^{\pi/4}\frac{t}{\tan t}\cdot\sec^2 t\,dt$

Again put $\tan t = v \Rightarrow \sec^2 t\,dt = dv$

$\therefore\ I = 2\displaystyle\int_0^1\frac{\tan^{-1}v}{v}dv = 2J \Rightarrow \frac{I}{J} = 2.$

14. (3) : Given $y = C_1(\cos x\cos C_2 - \sin x\sin C_2) - C_3e^{C_4}.e^{-x} + C_5\sin x$

$= (C_1\cos C_2)\cos x - (C_1\sin C_2 - C_5)\sin x - (C_3e^{C_4})e^{-x}$

$\Rightarrow\ y = l\cos x + m\sin x - ne^{-x}$, where l, m, n are arbitrary constants.

$\therefore$ Order = 3

15. (4) : The *d.r's* of the line are given by $l - m - n = 0$, $2l - m - 4n = 0$

$\Rightarrow\ l : m : n = 3 : 2 : 1$

A point on the line with $z = 0$ is given by

$x - y + 2 = 0, 2x - y + 6 = 0 \Rightarrow (x, y, z) = (-4, -2, 0)$

The line is $\dfrac{x+4}{3} = \dfrac{y+2}{2} = \dfrac{z}{1} = k$

Let $Q(3k - 4, 2k - 2, k)$ is a point on the line.

The *d.r's* of PQ are $3k - 8, 2k - 8, k - 2$

Since, PQ is perpendicular to the line.

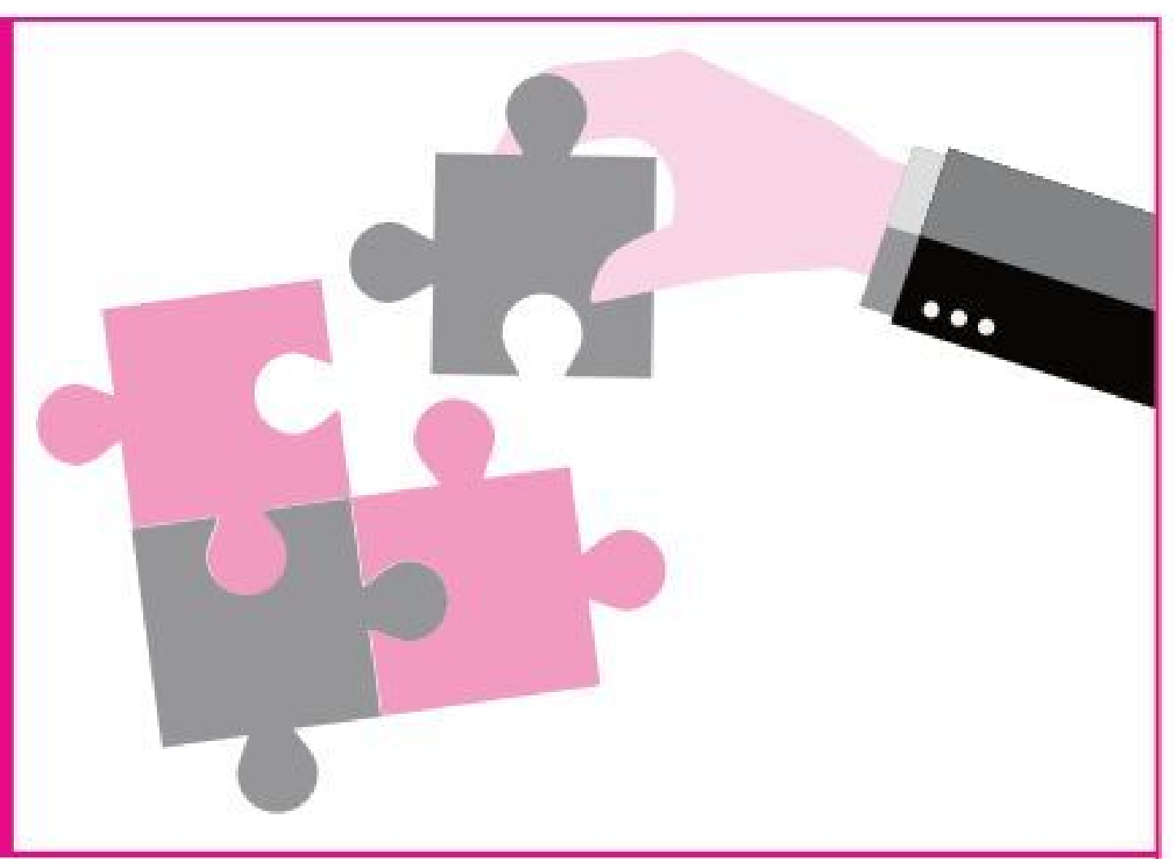
$\therefore\ 3(3k - 8) + 2(2k - 8) + k - 2 = 0 \Rightarrow k = 3$.

The foot is $Q(5, 4, 3)$.

Thus, $\dfrac{a+b+c}{3} = \dfrac{5+4+3}{3} = 4$.

◆◆

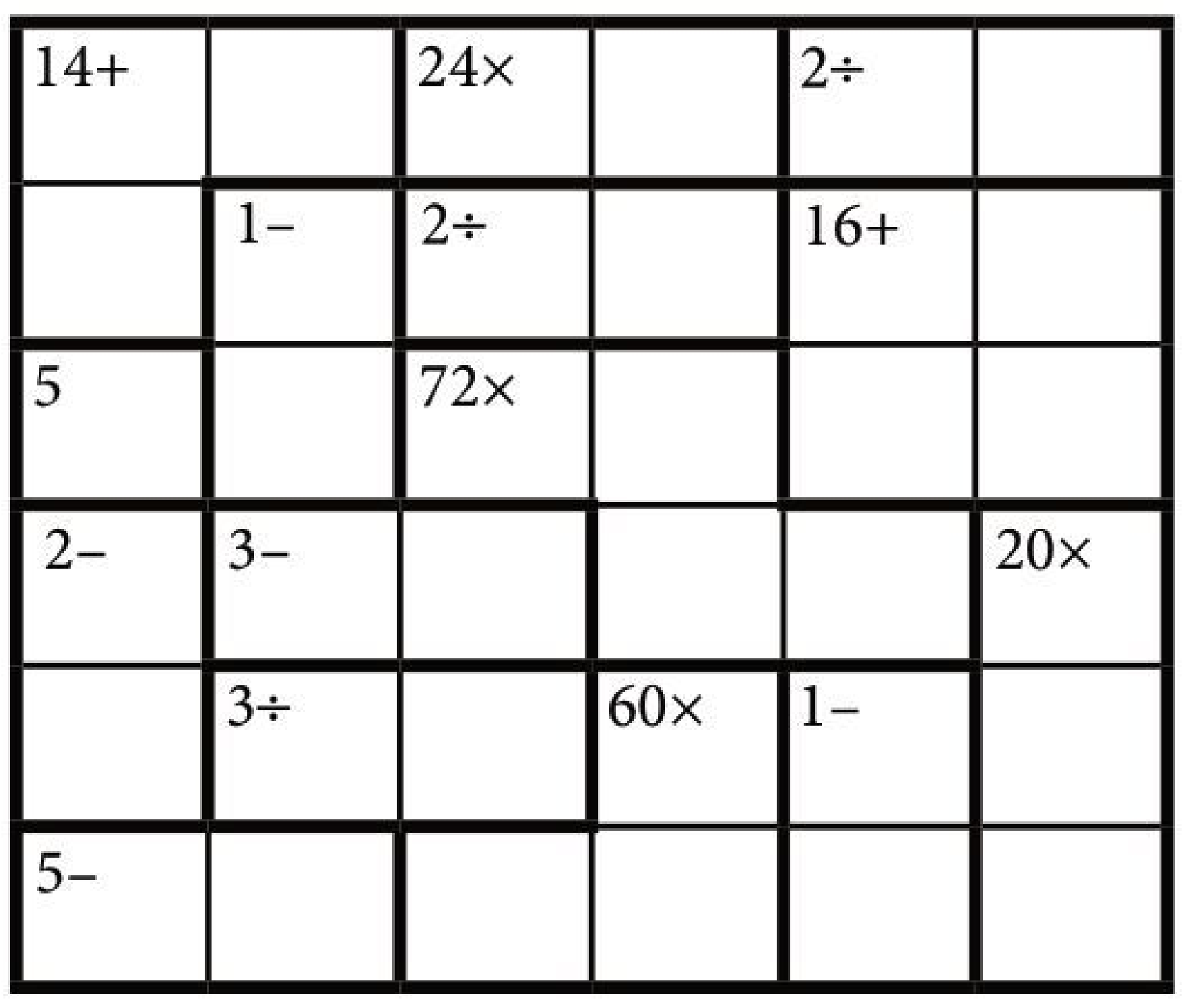
PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



14+		24×		2÷	
	1–	2÷		16+	
5		72×			
2–	3–				20×
	3÷		60×	1–	
5–					

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OLYMPIAD CORNER

1. Find the equation of the locus of a point P which moves so that the tangents from P to the circle $x^2 + y^2 = r^2$ cut off a line segment of length $2r$ on the line $x = r$.

2. Suppose that the function $f(u)$ has a second derivative in the interval (a, b), and that $f(u) \geq 0$ for all $u \in (a, b)$. Prove that

I. $(y - z) f(x) + (z - x) f(y) + (x - y) f(z) > 0$ for all $x, y, z \in (a, b)$, $z < y < x$ if and only if $f''(u) > 0$ for all $u \in (a, b)$.

II. $(y - z) f(x) + (z - x) f(y) + (x - y) f(z) = 0$ for all $x, y, z \in (a, b)$, $z < y < x$ if and only if $f(u)$ is a linear function on (a, b).

3. Compute tan 20° tan 40° tan 60° tan 80°.

4. Assume that a and b are integers. Prove that the equation $a^2 + b^2 + x^2 = y^2$ has an integer solution x, y if and only if the product ab is even.

5. Let ABC be an equilateral triangle and Γ its incircle. If D and E are points on AB and AC, respectively, such that DE is tangent to Γ, show that

$$\frac{AD}{DB} + \frac{AE}{EC} = 1.$$

6. Let b_n be the highest power of 3 dividing $\binom{3^k}{n}$, $0 \leq n \leq 3^k$. Calculate $\sum_{n=0}^{3^k} \frac{1}{b_n}$.

7. A sequence $\{x_n\}$ is defined by the rules $x_1 = 2$ and $nx_n = 2(2n - 1)x_{n-1}$; $n = 2, 3, \ldots$.
Prove that x_n is an integer for every positive integer n.

SOLUTIONS

1. From the circle, $x^2 + y^2 = a^2$, we find the points of intersection with the line $(y - q) = m(x - p)$.
Substituting $y = mx - mp + q$, we get
$x^2 + (mx - mp + q)^2 = a^2$, leading to the two solutions :

$$x = \frac{m(mp - q) \pm \sqrt{a^2(m^2 + 1) - m^2p^2 + q(2mp - q)}}{m^2 + 1}$$

For the line to be a tangent, the discriminant, $\Delta = a^2(m^2 + 1) - m^2p^2 + q(2mp - q)$, must be zero. So we solve $\Delta = 0$, to get

$$m = \frac{pq}{p^2 - a^2} \pm \frac{a\sqrt{p^2 + q^2 - a^2}}{p^2 - a^2}.$$

So, we have that the equations of the tangent lines from (p, q) to the circle $x^2 + y^2 = a^2$ are :

$$y = \left(-\frac{pq}{a^2 - p^2} \pm \frac{a\sqrt{a^2 - p^2 - q^2}}{p^2 - a^2} \right) x$$
$$- \left(-pq/(a^2 - p^2) \pm \frac{\sqrt{a^2 - p^2 - q^2}}{p^2 - a^2} \right) p + q.$$

Setting $x = a$ in these, and subtracting the two values leads to $(a + p)^2 = p^2 + q^2 - a^2$.
Thus the locus of P is given by $y^2 = 2a(a + x)$.
This is a parabola opening to the right with central axis, the x-axis and nose at $x = -a$. There are, ofcourse, three points that should be excluded : $(-a, 0)$, $(a, 2a)$ and $(a, -2a)$.

2. Part I is incorrect. The first statement is actually equivalent to "f is strictly convex", not "$f''(u) > 0$ for all $u \in (a, b)$". Strictly convex means that $f(tx + (1 - t)y) < t f(x) + (1 - t) f(y)$ for $x, y \in (a, b)$ and $0 < t < 1$. If f is twice differentiable this is equivalent to $f''(u) \geq 0$ everywhere with $f''(u) > 0$ on a dense set. For example, $f(x) = x^4$ is strictly convex everywhere but $f''(0) = 0$, and in this case

$$(y - z)x^4 + (z - x)y^4 + (x - y)z^4$$
$$= (x - z)(y - z)(x - y)\left(\frac{(x + y)^2 + (y + z)^2 + (x + z)^2}{2} \right) > 0$$

if $z < y < x$.

More generally, let f be any continuous function on (a, b) and $n \geq 3$ an integer. Given $x_1, x_2, ..., x_n$ let

$$L(f)=\sum_{i=1}^{n}(x_{i-1}-x_{i+1})f(x_i),$$

where we take $x_0 = x_n$ and $x_{n+1} = x_1$. Then the following are equivalent :

(i) $L(f) > 0$ whenever $a < x_1 < x_2 < ... < x_n < b$.

(ii) f is strictly convex on (a, b).

Similarly, for part II, we will show that f is linear on (a, b) if and only if $L(f) = 0$ whenever $a < x_1 < x_2 < ... < x_n < b$.

Proof of (i) $\Rightarrow$ (ii) : If f is not strictly convex, then either it is not convex at all or its graph includes a straight line segment. In the first case, there exist x, z and t with $a < z < x < b$ and $0 < t < 1$ such that

$f(tz + (1 - t)x) > tf(z) + (1 - t)f(x)$.

Taking $y = tz + (1 - t)x$ and nothing that

$t = (x - y)/(x - z)$ we can rewrite this as

$$f(y)>\frac{x-y}{x-z}f(z)+\frac{y-z}{x-z}f(x)$$

or $(y - z) f(x) + (z - x) f(y) + (x - y) f(z) < 0$. Let $x_1 = z$, $x_2 = y$ and $x_3 = ... = x_n = x$, and we have $L(f) < 0$ (if $n > 3$, note that the terms in $f(x_j)$ for $3 < j < n$ are 0 and the terms in $f(x_3)$ and $f(x_n)$ add to $(y - x) f(x)$). If $n > 3$ we must move $x_3, ..., x_n$ slightly so that $x_3 < ... < x_n$, but by continuity the inequality will still be true if the changes are small enough. In the second case, we can take any $x_1 < ... < x_n$ in the interval over which the graph is a straight line $f(x) = cx + d$, and then

$$L(f)=c\left(\sum_{i=1}^{n}x_{i-1}x_i-\sum_{i=1}^{n}x_ix_{i+1}\right)+d\left(\sum_{i=1}^{n}x_{i-1}-\sum_{i=1}^{n}x_{i+1}\right)=0$$

Proof of (ii) $\Rightarrow$ (i) : As noted above, if g is linear on (a, b) then $L(g) = 0$. Thus we can add a linear function g to f without changing either (i) or (ii).

With an appropriate choice of g we can make $f(x_1) = f(x_n) = 0$, and by strict convexity we then have $f(x_j) < 0$ for $1 < j < n$. Now write

$$L(f)=\sum_{i=1}^{n}(x_{i-1}-x_i)f(x_i)+\sum_{i=1}^{n}(x_i-x_{i+1})f(x_i)$$

$$=\sum_{i=1}^{n}(x_{i-1}-x_i)(f(x_i)+f(x_{i-1}))$$

Note that the $i = 1$ term is 0, and each of the other terms is strictly positive, so the sum is strictly positive.

As noted above, if f is linear on (a, b) then $L(f) = 0$. Conversely, suppose $L(f) = 0$ whenever $a < x_1 < x_2 < ... < x_n < b$. By part I, $L(h) > 0$ where $h(x) = x^2$, and so since L is a linear operator $L(f + ch) = cL(h) > 0$ for any $c > 0$. Again using part I, $f + ch$ is strictly convex, and so $f=\lim\limits_{c\to 0^+}(f+ch)$ is convex on (a, b). By the same argument, $-f$ is convex, so f is concave on (a, b). Any function that is both convex and concave on (a, b) is linear on (a, b).

3. Let $A = \tan 20° \tan 40° \tan 60° \tan 80°$. Then $A=\frac{B}{C}$,

where $B = 8 \sin 20° \sin 40° \sin 60° \sin 80°$

and $C = 8 \cos 20° \cos 40° \cos 60° \cos 80°$. Then

$B = 8 \sin 20° \sin 40° \sin 60° \sin 80°$

$= 2(\cos 60° + \cos 100°) (\cos 20° - \cos 100°)$

$= (1 + 2 \sin 10°) (\cos 20° + \sin 10°)$

$= \cos 20° + \sin 10° + 2 \sin 10° \cos 20° + 2 \sin^2 10°$

$= \cos 20° + \sin 10° + \sin 30° - \sin 10° + (1 - \cos 20°)$

$=\frac{3}{2}$,

$C = 8 \cos 20° \cos 40° \cos 60° \cos 80°$

$= 2(\cos 60° + \cos 100°) (\cos 20° + \cos 100°)$

$= (1 - 2 \sin 10°) (\cos 20° - \sin 10°)$

$= \cos 20° - \sin 10° - 2 \sin 10° \cos 20° + 2 \sin^2 10°$

$= \cos 20° - \sin 10° - \sin 30° + \sin 10° + (1 - \cos 20°)$

$=\frac{1}{2}$.

Therefore, $A=\frac{B}{C}=3$.

4. First, we prove that this condition is necessary. Suppose ab is odd. Then a, b are odd and $a^2 \equiv b^2 \equiv 1 \bmod 4$. Now $x^2 \equiv 0$ or $1 \bmod 4$, and $y^2 \equiv 0$ or $1 \bmod 4$. Therefore $a^2 + b^2 + x^2 = y^2$ is not possible, since if we consider the modulo 4, $2 + x^2 \equiv y^2 \bmod 4$, which is impossible since $2 + x^2 \equiv 2$ or $3 \bmod 4$. Therefore ab must be even.

If ab is even, then, without loss of generality, $a = 2k$.

Consider $4k^2 + b^2 + x^2 = y^2$.

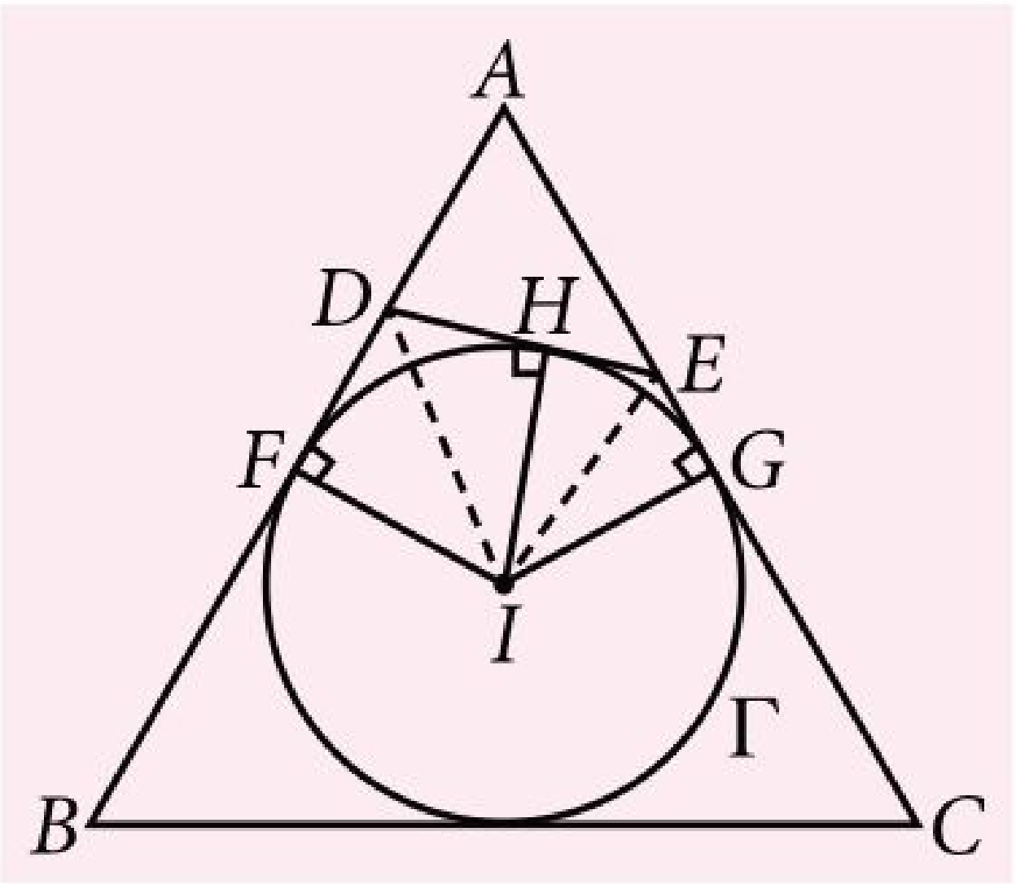
If $4k^2 + b^2 = 2t + 1$, t an integer, then set $y - x = 1$ and $y + x = 2t + 1$, $2y = (t + 1)2$, $y = t + 1$ and $x = t$.

Then $2t + 1 + t^2 = (t + 1)^2$. We are done.

If $4k^2 + b^2$ is even, then $b = 2s$ and $4k^2 + b^2 = 4(k^2 + s^2) = 4m$. Again, $y^2 - x^2 = 4m$.

Set $y - x = 2$ and $y + x = 2m$. Then $y = m + 1$ and $x = y - 2 = m - 1$.

Now $4m + (m - 1)^2 = (m + 1)^2$, and again we are done. Hence $a^2 + b^2 + x^2 = y^2$ always has a solution when ab is even.



5. Let I be the incentre and r the inradius. Let F, G, and H be the points where Γ is tangent to AB, AC, and DE. Then, $DF = DH$ and $EG = EH$ (this is a well-known property of the two tangents from a point to a circle). This implies that $\angle FID = \angle DIH$ and $\angle GIE = \angle EIH$. Since $\angle FIG = 120°$, $\angle FID + \angle GIE = 60°$. We can write $\angle FID = 30° + \phi$, and $\angle GIE = 30° - \phi$. Now, $FD = r\tan\angle FID$ and $AF = r\tan 60°$, so that

$$\frac{AD}{DB}=\frac{AF-FD}{AF+FD}=\frac{\tan 60°-\tan\angle FID}{\tan 60°+\tan\angle FID}$$

$$=\frac{\sqrt{3}-\dfrac{\tan 30°+\tan\phi}{1-\tan 30°\tan\phi}}{\sqrt{3}+\dfrac{\tan 30°+\tan\phi}{1-\tan 30°\tan\phi}}=\frac{\sqrt{3}-\dfrac{1+\sqrt{3}\tan\phi}{\sqrt{3}-\tan\phi}}{\sqrt{3}+\dfrac{1+\sqrt{3}\tan\phi}{\sqrt{3}-\tan\phi}}$$

$$=\frac{1-\sqrt{3}\tan\phi}{2},$$

and similarly,

$$\frac{AE}{EC}=\frac{1+\sqrt{3}\tan\phi}{2}$$

We then have

$$\frac{AD}{DB}+\frac{AE}{EC}=\frac{1-\sqrt{3}\tan\phi}{2}+\frac{1+\sqrt{3}\tan\phi}{2}=1$$

as required.

6. The exponent of the highest power of 3 dividing $t!$ is

$$\left[\frac{t}{3}\right]+\left[\frac{t}{3^2}\right]+\left[\frac{t}{3^3}\right]+\dots .$$

Thus, the exponent of the highest power of 3 dividing

$$\binom{3^k}{n}=\frac{3^k!}{n!(3^k-n)!} \text{ is } \sum_{j=1}^{k}\left(\left[\frac{3^k}{3^j}\right]-\left[\frac{n}{3^j}\right]-\left[\frac{3^k-n}{3^j}\right]\right)$$

Since $\dfrac{n}{3^j}+\dfrac{3^k-n}{3^j} = 3^{k-j}$, an integer,

$$\left[\frac{3^k}{3^j}\right]-\left[\frac{n}{3^j}\right]-\left[\frac{3^k-n}{3^j}\right]$$

can only take two possible values : 0 and 1.

When $3^j \mid n$, the value is 0. Otherwise, it is 1. Therefore, the value of b_n is 3^k divided by the highest power of 3 dividing n.

Consider the numbers divisible by 3^j, $0 \le j \le k-1$. Ignoring 0, we see that 3^{k-j} of the numbers are divisible by 3^j. These numbers can be written as $3^j \cdot (3m)$, $3^j \cdot (3m+1)$, or $3^j \cdot (3m+2)$. Therefore, there are $2 \cdot 3^{k-j-1}$ numbers in $\{1, 2, \dots, 3^k\}$ for which 3^j is the highest power of 3 dividing them.

Each of the $2 \cdot 3^{k-j-1}$ numbers contributes $\dfrac{1}{3^{k-j}}$ to the sum $\displaystyle\sum_{n=0}^{3^k}\frac{1}{b_n}$ for a total of $\dfrac{2}{3}$. Therefore,

$$\sum_{n=0}^{3^k}\frac{1}{b_n}=\sum_{n=1}^{3^k-1}\frac{1}{b_n}+2=\frac{2}{3}k+2.$$

7. We have, $x_2 = 2 \cdot \dfrac{3}{2}\, x_1$

$$x_3 = 2 \cdot \frac{5}{3}\, x_2$$

....

$$x_{n-1} = 2 \cdot \frac{2n-3}{n-1}\, x_{n-2}$$

$$x_n = 2 \cdot \frac{2n-1}{n}\, x_{n-1}$$

It follows that

$$x_n = 2^{n-1}\frac{(2n-1)(2n-3)\dots 5\cdot 3}{n\cdot(n-1)\dots 3\cdot 2}\cdot 2=\frac{(2n)!}{(n!)^2}=\binom{2n}{n}\in\mathbb{Z}$$

since $\dbinom{2n}{n}$ is a binomial coefficient.

SAMURAI SUDOKU

SOLUTION - NOVEMBER 2021

8	6	2	3	4	7	5	1	9				6	1	8	4	2	5	3	7	9
3	7	1	6	9	5	4	8	2				4	7	9	8	3	1	5	2	6
5	9	4	1	2	8	6	3	7				3	2	5	9	6	7	4	8	1
1	2	6	9	5	3	8	7	4				7	4	2	3	9	6	8	1	5
7	5	3	8	1	4	2	9	6				9	8	1	2	5	4	7	6	3
9	4	8	7	6	2	1	5	3				5	3	6	7	1	8	9	4	2
4	8	9	2	7	1	3	6	5	7	8	2	1	9	4	6	7	3	2	5	8
6	3	5	4	8	9	7	2	1	4	9	5	8	6	3	5	4	2	1	9	7
2	1	7	5	3	6	9	4	8	6	3	1	2	5	7	1	8	9	6	3	4
						5	7	3	8	1	4	6	2	9						
						1	9	2	3	6	7	5	4	8						
						4	8	6	2	5	9	7	3	1						
5	9	1	2	4	6	8	3	7	9	2	6	4	1	5	7	2	3	8	6	9
3	8	6	9	5	7	2	1	4	5	7	3	9	8	6	5	1	4	3	7	2
2	7	4	3	8	1	6	5	9	1	4	8	3	7	2	8	6	9	1	5	4
7	3	9	8	6	5	1	4	2				8	3	9	2	5	7	6	4	1
6	1	5	4	9	2	3	7	8				6	5	1	3	4	8	2	9	7
4	2	8	7	1	3	5	9	6				7	2	4	1	9	6	5	8	3
1	5	7	6	2	9	4	8	3				5	4	3	9	8	1	7	2	6
9	4	2	5	3	8	7	6	1				2	9	7	6	3	5	4	1	8
8	6	3	1	7	4	9	2	5				1	6	8	4	7	2	9	3	5

YOU ASK WE ANSWER

Do you have a question that you just can't get answered? Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. If $u=\sqrt{z_1z_2}$, then prove that

$$|z_1|+|z_2|=\left|\frac{z_1+z_2}{2}+u\right|+\left|\frac{z_1+z_2}{2}-u\right|.$$

(Mukul Paul, Punjab)

Ans. $\text{R.H.S.}=\left|\frac{z_1+z_2}{2}+\sqrt{z_1z_2}\right|+\left|\frac{z_1+z_2}{2}-\sqrt{z_1z_2}\right|$

$=\frac{1}{2}\left|z_1+z_2+2\sqrt{z_1z_2}\right|+\frac{1}{2}\left|z_1+z_2-2\sqrt{z_1z_2}\right|$

$=\frac{1}{2}\left|(\sqrt{z_1}+\sqrt{z_2})^2\right|+\frac{1}{2}\left|(\sqrt{z_1}-\sqrt{z_2})^2\right|$

$=\frac{1}{2}\left|\sqrt{z_1}+\sqrt{z_2}\right|^2+\frac{1}{2}\left|\sqrt{z_1}-\sqrt{z_2}\right|^2$

$[\because |z^2|=|z\cdot z|=|z|\,|z|=|z|^2]$

Now, for any two complex numbers α, β, we get

$|\alpha+\beta|^2+|\alpha-\beta|^2$

$=(\alpha+\beta)\overline{(\alpha+\beta)}+(\alpha-\beta)\overline{(\alpha-\beta)}$ $\quad [\because |z|^2=z\bar{z}]$

$=(\alpha+\beta)(\bar{\alpha}+\bar{\beta})+(\alpha-\beta)(\bar{\alpha}-\bar{\beta})$

$[\because \overline{(z_1\pm z_2)}=\bar{z}_1\pm\bar{z}_2]$

$=\alpha\bar{\alpha}+\alpha\bar{\beta}+\beta\bar{\alpha}+\beta\bar{\beta}+\alpha\bar{\alpha}-\alpha\bar{\beta}-\beta\bar{\alpha}+\beta\bar{\beta}$

$=2|\alpha|^2+2|\beta|^2$

$\therefore\quad \frac{1}{2}|\alpha+\beta|^2+\frac{1}{2}|\alpha-\beta|^2=|\alpha|^2+|\beta|^2$

So, $\text{R.H.S.}=|\sqrt{z_1}|^2+|\sqrt{z_2}|^2=|(\sqrt{z_1})^2|+|(\sqrt{z_2})^2|$

$=|z_1|+|z_2|=\text{L.H.S.}$

2. Solve the differential equation :

$$\frac{dy}{dx}+\frac{(ax+hy+g)}{(hx+by+f)}=0$$

(Satyadev, Uttar Pradesh)

Ans. Given differential equation is

$$\frac{dy}{dx}+\frac{ax+hy+g}{hx+by+f}=0$$

$\Rightarrow\quad (hx+by+f)dy+(ax+hy+g)dx=0$

$\Rightarrow\quad hx\,dy+by\,dy+f\,dy+ax\,dx+hy\,dx+gdx=0$

$\Rightarrow\quad h(x\,dy+y\,dx)+by\,dy+f\,dy+ax\,dx+gdx=0$

$\Rightarrow\quad h\,d(xy)+by\,dy+f\,dy+ax\,dx+gdx=0$

On integrating both sides, we get

$h\,xy+b\frac{y^2}{2}+fy+a\frac{x^2}{2}+gx=c$, which is the required solution.

3. If $a<b<c<d$, then prove that for any real λ, the quadratic equation $(x-a)(x-c)+\lambda(x-b)(x-d)=0$ has real roots. ***(Aniket, West Bengal)***

Ans. The given equation reduces to

$(1+\lambda)x^2-\{(a+c)+\lambda(b+d)\}x+(ac+\lambda bd)=0$ $\quad$...(i)

Now, if the roots are real, then discriminant ≥ 0

i.e., $\{a+c+\lambda(b+d)\}^2-4(1+\lambda)(ac+\lambda bd)\geq 0$

i.e., $\{(b+d)^2-4bd\}\lambda^2+\{2(a+c)(b+d)-4(ac+bd)\}\lambda+\{(a+c)^2-4ac\}\geq 0$

i.e., $(b-d)^2\lambda^2+2\{(a+c)(b+d)-2(ac+bd)\}\lambda+(a-c)^2\geq 0$ $\quad$...(ii)

The discriminant of quadratic expression (ii) is

$D=4[(a+c)(b+d)-2(ac+bd)]^2-4(b-d)^2(a-c)^2$

$=4[ab+bc+ad+cd-2ac-2bd+(b-d)(a-c)]$

$\times[ab+bc+ad+cd-2ac-2bd-(b-d)(a-c)]$

$[\because x^2-y^2=(x+y)(x-y)]$

$=4[2ab+2cd-2ac-2bd][2bc+2ad-2ac-2bd]$

$=16[ab+cd-ac-bd][bc+ad-ac-bd]$

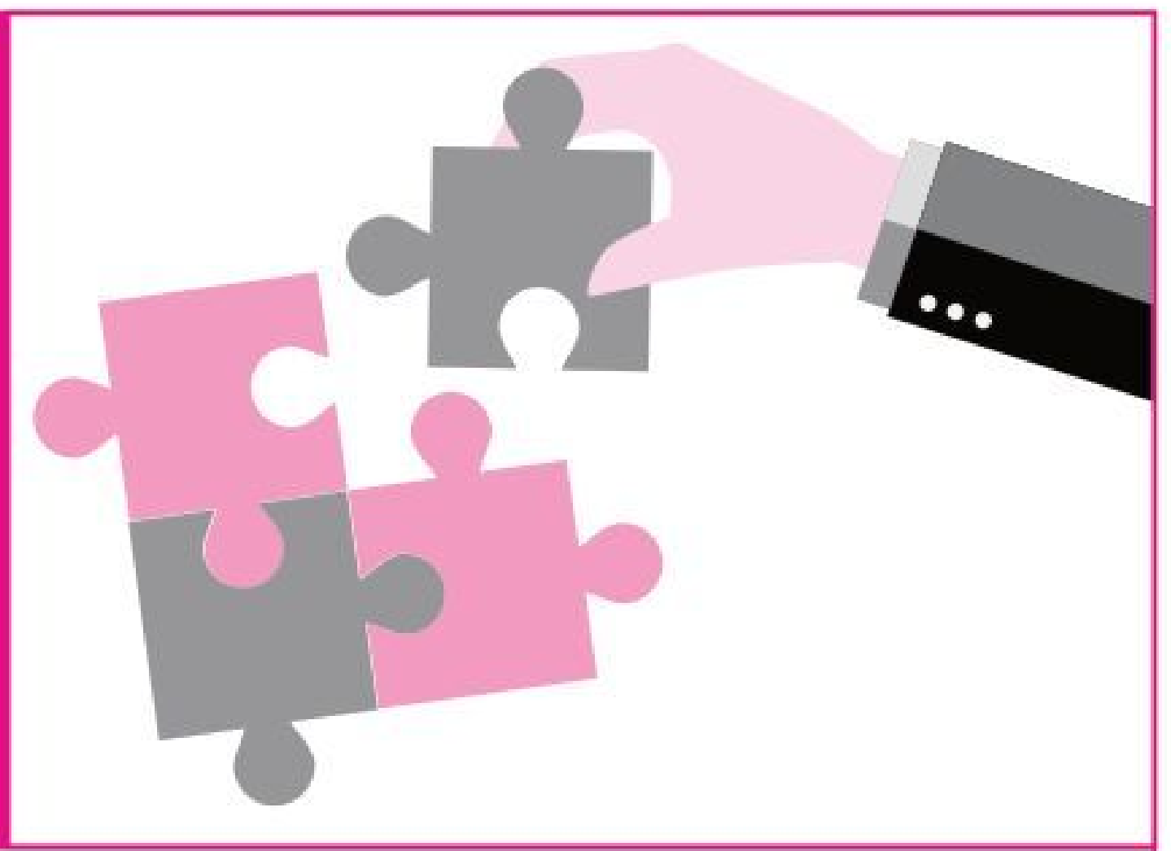
$=16(a-b)(b-c)(c-d)(d-a)<0$ $\quad [\because a<b<c<d]$

Also, coeff. of $\lambda^2=(b-d)^2$, which is +ve.

Hence, inequality (ii) is true for any real λ which proves that equation (i) has real roots for any real λ.

◆ ◆

PUZZLE CORNER



SOLUTION · NOVEMBER 2021

6× 2	3	1− 5	4	4− 1	3÷ 6
1	48× 4	14+ 3	6	5	2
15+ 6	2	5+ 4	5	3÷ 3	1
4	6	1	1− 3	10× 2	5
5	5− 1	6	2	1− 4	3
2− 3	5	2÷ 2	1	2− 6	4

CBSE warm-up!

CLASS-XI

TERM-I **OBJECTIVE TYPE QUESTIONS**

Practice Paper 2021

Time allowed : 90 minutes
Maximum marks : 40

General Instructions :

1. *This question paper contains three sections A, B and C. Each part is compulsory.*
2. *Section - A has 20 MCQs, attempt any 16 out of 20.*
3. *Section - B has 20 MCQs, attempt any 16 out of 20*
4. *Section - C has 10 MCQs, attempt any 8 out of 10.*
5. *There is no negative marking.*
6. *All questions carry equal marks.*

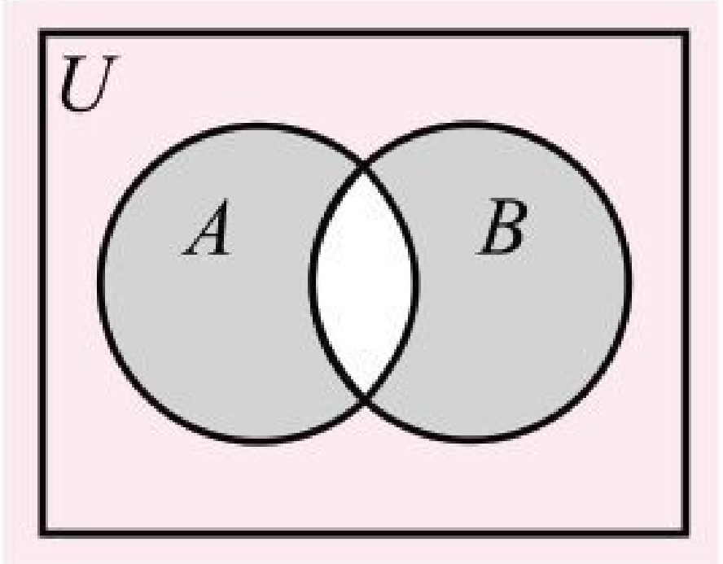
Mark your answer in the Sample OMR Sheet given after Practice Paper to get familiar with the actual paper pattern.

SECTION-A

In this section, attempt any 16 questions out of Questions 1-20. Each Question is of 1 mark weightage.

1. The shaded region in the figure represents

(a) $A \cap B$
(b) $A \cup B$
(c) $A \subset B$
(d) none of these



2. Let $A = \{x : x^2 - 5x + 6 = 0\}$, $B = \{2, 4\}$, $C = \{4, 5\}$, then $A \times (B \cap C)$ is

(a) $\{(2, 4), (3, 4)\}$ (b) $\{(4, 2), (4, 3)\}$
(c) $\{(2, 4), (3, 4), (4, 4)\}$
(d) $\{(2, 2), (3, 3), (4, 4), (5, 5)\}$

3. The real values of x and y for which the following equality hold, are respectively

$(x^4 + 2xi) - (3x^2 + iy) = (3 - 5i) + (1 + 2iy)$

(a) 2, 3 or –2, 1/3 (b) 1, 3 or –1, 1/3
(c) 2, 1/3 or –2, 3 (d) 2, 1/3 or –2, –1/3

4. If $1 - i$ is a root of the equation $x^2 + ax + b = 0$, where $a, b \in R$, then find the values of a and b.

(a) 2, 2 (b) –2, 2 (c) –2, –2 (d) 1, 2

5. The range of the following set of observations 2, 3, 5, 9, 8, 7, 6, 5, 7, 4, 3 is

(a) 11 (b) 7 (c) 5.5 (d) 6

6. If the lines $2y = x + 3$ and $y = 3x + 1$ are equally inclined to the line, $y = mx + 4$, then the value of m is

(a) $\frac{1 \pm 5\sqrt{2}}{7}$ (b) $\frac{5\sqrt{2} - 1}{7}$
(c) $\frac{1 - 5\sqrt{2}}{5}$ (d) None of these

7. If the set A contains 5 elements, then the number of elements in the power set $P(A)$ is

(a) 32 (b) 25 (c) 16 (d) 8

8. If number of elements in sets A and B are m and n respectively, then the number of relations from A to B is

(a) 2^{m+n} (b) 2^{mn} (c) $m + n$ (d) mn

9. $\lim_{x \to -1} [1 + x + x^2 + \dots + x^{10}]$ is equal to

(a) 0 (b) 1
(c) 2 (d) does not exist

10. The mean deviation of the data 2, 9, 9, 3, 6, 9, 4 from the mean is approximately
(a) 2.23 (b) 2.57 (c) 3.23 (d) 3.57

11. The value of $x^3 + 7x^2 - x + 16$ when $x = 1 + 2i$, is
(a) $17 - 24i$ (b) $-17 + 24i$
(c) $-17 - 24i$ (d) $17 + 24i$

12. Find the equation of the straight line passing through (1, 2) and perpendicular to the line $x + y + 7 = 0$.
(a) $x + y + 1 = 0$ (b) $x + y - 1 = 0$
(c) $x - y - 1 = 0$ (d) $x - y + 1 = 0$

13. Solve : $x^2 + 2 = 0$
(a) $\pm\sqrt{2}\,i$ (b) $\pm i$ (c) $\pm\sqrt{3}\,i$ (d) $\pm 2i$

14. If p is the length of perpendicular from origin to the line whose intercepts on the axes are a and b, then $\frac{1}{a^2}+\frac{1}{b^2}$ is equal to
(a) p^2 (b) $\frac{1}{p^2}$ (c) $2p^2$ (d) $\frac{1}{2p^2}$

15. Which of the following has only one subset?
(a) { } (b) {4} (c) {4, 5} (d) {0}

16. If $f : R \to R$ is defined by $f(x)=\frac{x}{x^2+1}$, $f(f(2))$ is equal to
(a) 29/9 (b) 29/8 (c) 29/10 (d) 10/29

17. $\lim\limits_{x\to 2a} \frac{\sqrt{x-2a}+\sqrt{x}-\sqrt{2a}}{\sqrt{x^2-4a^2}}$ is equal to
(a) $\frac{1}{\sqrt{a}}$ (b) $\frac{1}{2\sqrt{a}}$ (c) $\frac{\sqrt{a}}{2}$ (d) $\frac{1}{2a}$

18. Find the distance of the point (3, –5) from the line $3x - 4y - 26 = 0$.
(a) $\frac{2}{5}$ unit (b) 4 units
(c) $\frac{3}{5}$ unit (d) 6 units

19. The mean and variance of n observations $x_1, x_2, x_3, \ldots, x_n$ are 5 and 0 respectively. If $\sum_{i=1}^{n} x_i^2 = 400$, then the value of n is equal to
(a) 80 (b) 25 (c) 20 (d) 16

20. If the variance of 1, 2, 3, 4, 5, …, 10 is $\frac{99}{12}$, then the standard deviation of 3, 6, 9, 12, …, 30 is
(a) $\frac{297}{4}$ (b) $\frac{3}{2}\sqrt{33}$ (c) $\frac{3}{2}\sqrt{99}$ (d) $\sqrt{\frac{99}{12}}$

SECTION-B

In this section, attempt any 16 questions out of the Questions 21 - 40. Each Question is of 1 mark weightage.

21. $\lim\limits_{x\to 0} \frac{\sin 3x}{\sin 2x} =$
(a) 1 (b) 1/2 (c) 3/2 (d) 0

22. The set of all integers 'x' such that $|x - 3| < 2$ is equal to
(a) {1, 2, 3, 4, 5} (b) {1, 2, 3, 4}
(c) {2, 3, 4} (d) {–4, –3, –2}

23. If $f(x) = x^2 - x^{-2}$, $x \in R - \{0\}$, then $f(1/x)$ is equal to
(a) $f(x)$ (b) $-f(x)$ (c) $1/f(x)$ (d) $(f(x))^2$

24. $\lim\limits_{x\to \frac{\pi}{2}} \frac{1-\cos x}{\sin x}$ is equal to
(a) 0 (b) –1
(c) 1 (d) Does not exist

25. Express $(-\sqrt{3}+\sqrt{-2})(2\sqrt{3}-i)$ in the form $a + ib$.
(a) $(1 - 2\sqrt{2})i$ (b) $(1 + 2\sqrt{2})i$
(c) $(-6-\sqrt{2})+\sqrt{3}(1+2\sqrt{2})i$
(d) $(-6+\sqrt{2})+\sqrt{3}(1+2\sqrt{2})i$

26. Solve : $x^2 + x + 1 = 0$
(a) $\pm i$ (b) $\pm(1/2)i$
(c) $\frac{-1\pm\sqrt{3}i}{2}$ (d) $\frac{\pm\sqrt{3}i}{2}$

27. Find the mean deviation about the mean for the following data :
12, 3, 18, 17, 4, 9, 17, 19, 20, 15, 8, 17, 2, 3, 16, 11, 3, 1, 0, 5
(a) 8.2 (b) 9.2 (c) 6.2 (d) 5.2

28. The line passing through (1, 1) and parallel to the line $2x - 3y + 5 = 0$ is
(a) $3x + 2y = 5$ (b) $2x - 3y + 1 = 0$
(c) $3x - 2y = 1$ (d) $2x + 3y = 5$

29. If the range of the given data arranged in descending order is 148, then the value of x is ________.
x, 178, 146, 108, 105, 100, 100, 98, 96
(a) 200 (b) 250 (c) 244 (d) 190

30. Which of the following is a finite set?
(a) Set of natural numbers N
(b) Set of rational numbers Q
(c) Number of days in a week
(d) Set of concentric circles

31. The domain of the function $f(x) = \frac{x^2+3x+5}{x^2-5x+4}$ is
(a) R (b) $R - \{1, 4\}$
(c) $R - \{1\}$ (d) (1, 4)

32. Solve : $\sqrt{5}x^2+x+\sqrt{5}=0$

(a) $\pm\frac{\sqrt{19}}{5}i$ (b) $\pm\frac{\sqrt{19}i}{2}$

(c) $\frac{-1\pm\sqrt{19}i}{2\sqrt{5}}$ (d) $\frac{-1\pm\sqrt{19}i}{\sqrt{5}}$

33. The equation of the perpendicular bisector of the line segment joining $A(-2, 3)$ and $B(6, -5)$ is

(a) $x-y=-1$ (b) $x-y=3$

(c) $x+y=3$ (d) $x+y=1$

34. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}=e^2$, then

(a) $a=2, b=1$ (b) $a=1, b=2$

(c) $a=1, b\in R$ (d) $a=b=1$

35. The equation of smallest degree with real coefficients having $2+3i$ as one of the roots is

(a) $x^2+4x+13=0$ (b) $x^2+4x-13=0$

(c) $x^2-4x+13=0$ (d) $x^2-4x-13=0$

36. If $n(A)=12$, $n(B)=8$, $n(A\cap B)=4$, then $n(A\cup B)$ is equal to

(a) 20 (b) 12 (c) 16 (d) 18

37. $\lim_{x\to 0}\frac{\tan x-\sin x}{\sin^3 x}$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) 0 (d) Does not exist

38. Find the angle between the x-axis and the line joining the points $(3, -1)$ and $(4, -2)$.

(a) 130° (b) 135°

(c) 150° (d) None of these

39. The arg $\left(\frac{3+i}{2-i}+\frac{3-i}{2+i}\right)$ is equal to

(a) $\pi/2$ (b) 0 (c) $\frac{\pi}{4}$ (d) $-\frac{\pi}{4}$

40. Find the mean deviation about the median for the data given below:

11, 3, 8, 7, 5, 14, 10, 2, 9

(a) 3 (b) 2 (c) 5 (d) 4

SECTION-C

- *In this section, attempt any 8 questions.*
- *Each question is of 1-mark weightage.*
- *Questions 46-50 are based on a Case-Study.*

41. If A and B are subsets of universal set U such that $n(U)=800$, $n(A)=300$, $n(B)=400$ and $n(A\cap B)=100$. The number of elements in the set $A\cup B$ is

(a) 400 (b) 600 (c) 500 (d) 300

42. The domain for which the functions $f(x)=\frac{1}{3}\left(x^2-\frac{1}{2}\right)$ and $g(x)=\frac{1}{2}\left(\frac{1}{3}-x\right)$ are equal, is

(a) R (b) $\left\{-2,\frac{1}{2}\right\}$ (c) R^+ (d) $\left\{2,\frac{1}{2}\right\}$

43. $i^2+i^4+i^6+\ldots.(2n+1)$ terms =

(a) 1 (b) -1 (c) i (d) $-i$

44. If the slope of the line joining the points (3, 4) and $(-2, a)$ is equal to $-\frac{2}{5}$, then the value of a is equal to

(a) 6 (b) 4 (c) 3 (d) 2

45. Find the standard deviation of 12, 23, 34, 45, 56, 67 and 78.

(a) 2 (b) 4 (c) 22 (d) 11

CASE STUDY

Jashanpreet applied for the teaching job and got selected. She has been offered the job with certain salary along with certain annual increment. In 2nd year, she was receiving ₹ 13750 as monthly salary and in 5th years, she was receiving ₹ 17500 as monthly salary. Assume that annual increment is fixed by a constant amount.

Based on the above information, answer the following questions.

46. The fixed amount by which salary is increasing every year, is

(a) ₹1100 (b) ₹1250 (c) ₹1200 (d) ₹1150

47. Initial salary of Jashanpreeet is

(a) ₹12500 (b) ₹1100 (c) ₹12000 (d) ₹11200

48. Find the monthly salary of Jashanpreet in 10th year.

(a) ₹ 22570 (b) ₹ 24000

(c) ₹ 23750 (d) ₹ 25000

49. Total amount received by Jashanpreet in 10 years is

(a) ₹ 3075000 (b) ₹ 2500000

(c) ₹ 2750000 (d) ₹ 2175000

50. If S_1, S_2 and S_3 are the sum of n, $2n$ and $3n$ terms respectively of an A.P., then $\frac{S_3}{S_2-S_1}$, is equal to

(a) 1 (b) 2 (c) 3 (d) 4

SAMPLE OMR SHEET

General Instructions

- *Details of the candidates will be pre-filled in the OMR.*
- *Candidate has to write a Question Paper Code in the upper right corner in the space provided for the same. Question Paper Code will be written on the Question Paper.*
- *Candidate has to write in his/her own hand in the space provided **"I confirm that all particulars given above are correct"** and sign in the running hand on OMR.*
- *Only Blue/Black ball point pen is to be used for filling desired information and for giving responses.*
- *Use of pencil is strictly prohibited.*

Important features of the OMR:

- *There is a space for answering 60 questions in an OMR irrespective of total questions given in the Question Papers of any subject.*
- *Answers given after the maximum question number in your question paper will not be evaluated.*

उत्तर पत्रिका / ANSWER SHEET
केवल नीले/काले बॉल प्वाइंट पेन का ही प्रयोग करें।
USE ONLY BLUE/BLACK BALL POINT PEN

उच्च माध्यमिक विद्यालय परीक्षा (कक्षा ग्यारहवीं)
Senior Secondary School Examination (Class XI)

अनुक्रमांक **Roll No.**	
(अंकों में एवं शब्दों में) (in digits and words)	
परीक्षा का दिन एवं तिथि Day & Date of the Examination	
केन्द्र संख्या व नाम Center No. & Name	
विषय कोड व नाम Subject code & Name	041 & Mathematics
अभ्यार्थी का नाम Candidate's Name	
पिता का नाम Father's Name	
विद्यालय का कोड व नाम School code and Name	

अधीक्षक की मोहर / Facsimile stamp of the Center Supdt.

OMR No.

प्रश्न पत्र के ऊपर लिखे कोड को दर्शाएं
Write code No. written on the question paper
Question Paper Code

☐☐☐ / ☐☐ / ☐

SOME EXAMPLES OF WRONG/CORRECT WAYS OF MARKING AS FOLLOWS

WRONG METHOD					CORRECT METHOD				
1	✓	☒	ⓒ	ⓓ	1	●	ⓑ	ⓒ	ⓓ
2	ⓐ	✓	ⓒ	ⓓ	2	ⓐ	●	ⓒ	ⓓ
3	ⓐ	ⓑ	✓	●	3	ⓐ	ⓑ	●	ⓓ
4	ⓐ	ⓑ	ⓒ	▣	4	ⓐ	ⓑ	ⓒ	●
5	●	☒	ⓒ	ⓓ	5	●	ⓑ	ⓒ	ⓓ
6	ⓐ	ⓑ	☒	ⓓ	6	ⓐ	ⓑ	ⓒ	ⓓ
7	ⓐ	ⓑ	◒	ⓓ	7	ⓐ	ⓑ	●	ⓓ
8	ⓐ	⊖	ⓒ	●	8	ⓐ	ⓑ	ⓒ	●

प्रत्येक प्रश्न का उत्तर **केवल एक ही गोले में** गहरा निशान लगाकर दीजिए एवं उत्तर को बोक्स में लिखें Darken **ONLY ONE CIRCLE** for each Question and write the answer in the box.

Q.No.	Response	*	#	Q.No.	Response	*	#	Q.No.	Response	*	#	Q.No.	Response	*	#
1	ⓐ ⓑ ⓒ ⓓ		①	16	ⓐ ⓑ ⓒ ⓓ		⑯	31	ⓐ ⓑ ⓒ ⓓ		㉛	46	ⓐ ⓑ ⓒ ⓓ		㊻
2	ⓐ ⓑ ⓒ ⓓ		②	17	ⓐ ⓑ ⓒ ⓓ		⑰	32	ⓐ ⓑ ⓒ ⓓ		㉜	47	ⓐ ⓑ ⓒ ⓓ		㊼
3	ⓐ ⓑ ⓒ ⓓ		③	18	ⓐ ⓑ ⓒ ⓓ		⑱	33	ⓐ ⓑ ⓒ ⓓ		㉝	48	ⓐ ⓑ ⓒ ⓓ		㊽
4	ⓐ ⓑ ⓒ ⓓ		④	19	ⓐ ⓑ ⓒ ⓓ		⑲	34	ⓐ ⓑ ⓒ ⓓ		㉞	49	ⓐ ⓑ ⓒ ⓓ		㊾
5	ⓐ ⓑ ⓒ ⓓ		⑤	20	ⓐ ⓑ ⓒ ⓓ		⑳	35	ⓐ ⓑ ⓒ ⓓ		㉟	50	ⓐ ⓑ ⓒ ⓓ		㊿
6	ⓐ ⓑ ⓒ ⓓ		⑥	21	ⓐ ⓑ ⓒ ⓓ		㉑	36	ⓐ ⓑ ⓒ ⓓ		㊱	51	ⓐ ⓑ ⓒ ⓓ		(51)
7	ⓐ ⓑ ⓒ ⓓ		⑦	22	ⓐ ⓑ ⓒ ⓓ		㉒	37	ⓐ ⓑ ⓒ ⓓ		㊲	52	ⓐ ⓑ ⓒ ⓓ		(52)
8	ⓐ ⓑ ⓒ ⓓ		⑧	23	ⓐ ⓑ ⓒ ⓓ		㉓	38	ⓐ ⓑ ⓒ ⓓ		㊳	53	ⓐ ⓑ ⓒ ⓓ		(53)
9	ⓐ ⓑ ⓒ ⓓ		⑨	24	ⓐ ⓑ ⓒ ⓓ		㉔	39	ⓐ ⓑ ⓒ ⓓ		㊴	54	ⓐ ⓑ ⓒ ⓓ		(54)
10	ⓐ ⓑ ⓒ ⓓ		⑩	25	ⓐ ⓑ ⓒ ⓓ		㉕	40	ⓐ ⓑ ⓒ ⓓ		㊵	55	ⓐ ⓑ ⓒ ⓓ		(55)
11	ⓐ ⓑ ⓒ ⓓ		⑪	26	ⓐ ⓑ ⓒ ⓓ		㉖	41	ⓐ ⓑ ⓒ ⓓ		㊶	56	ⓐ ⓑ ⓒ ⓓ		(56)
12	ⓐ ⓑ ⓒ ⓓ		⑫	27	ⓐ ⓑ ⓒ ⓓ		㉗	42	ⓐ ⓑ ⓒ ⓓ		㊷	57	ⓐ ⓑ ⓒ ⓓ		(57)
13	ⓐ ⓑ ⓒ ⓓ		⑬	28	ⓐ ⓑ ⓒ ⓓ		㉘	43	ⓐ ⓑ ⓒ ⓓ		㊸	58	ⓐ ⓑ ⓒ ⓓ		(58)
14	ⓐ ⓑ ⓒ ⓓ		⑭	29	ⓐ ⓑ ⓒ ⓓ		㉙	44	ⓐ ⓑ ⓒ ⓓ		㊹	59	ⓐ ⓑ ⓒ ⓓ		(59)
15	ⓐ ⓑ ⓒ ⓓ		⑮	30	ⓐ ⓑ ⓒ ⓓ		㉚	45	ⓐ ⓑ ⓒ ⓓ		㊺	60	ⓐ ⓑ ⓒ ⓓ		(60)

SAMPLE

***** **Write Correct Option** **#** **Darken Circle if question not attempted**

I confirm that all particulars given above are correct.

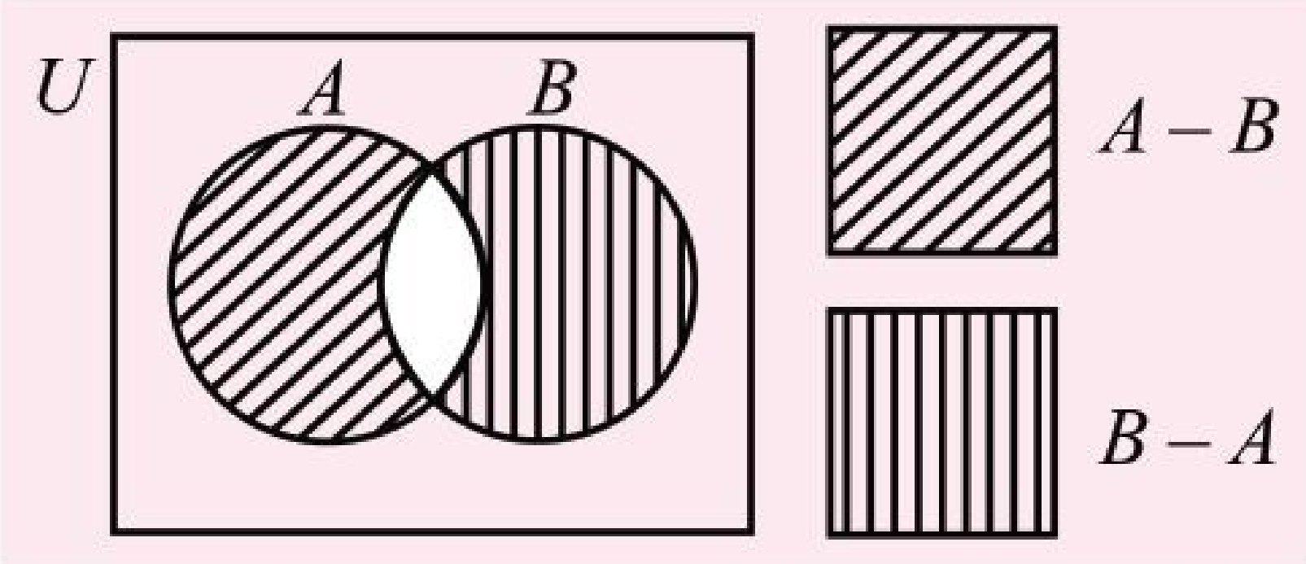
अभ्यर्थी के हस्ताक्षर (चलते हाथ से लिखे)
Candidate's Signature (in running hand)

Total correct answers **(To be filled by the Evaluator)**			**Total correct answers** **(To be filled by the Coordinator)**		

For more details visit cbse.gov.in

SOLUTIONS

1. **(d) :**



$\Rightarrow$ Shaded part is given by $(A - B) \cup (B - A)$

2. **(a) :** Given, $A = \{x : x^2 - 5x + 6 = 0\}$

Clearly, $x^2 - 5x + 6 = 0 \Rightarrow (x - 3)(x - 2) = 0 \Rightarrow x = 3, 2$

$\therefore$ $A = \{3, 2\}$. Also, we have $B = \{2, 4\}$ and $C = \{4, 5\}$

$\therefore$ $B \cap C = \{4\} \Rightarrow A \times (B \cap C)$

$= \{3, 2\} \times \{4\} = \{(3, 4), (2, 4)\}$

3. **(a) :** The given equality can be rewritten as

$(x^4 - 3x^2) + i(2x - y) = 4 + i(2y - 5)$

$\Rightarrow$ $x^4 - 3x^2 = 4,\ 2x - y = 2y - 5$

$\Rightarrow$ $(x^2 - 4)(x^2 + 1) = 0 \Rightarrow x = \pm 2 \qquad (\because x^2 \neq -1)$

$\therefore$ At $x = 2$, $y = 3$ and at $x = -2$, $y = 1/3$

4. **(b) :** Since complex roots always occur in conjugate pair.

$\therefore$ Other root is $1 + i$.

Sum of roots $= \dfrac{-a}{1} = (1 - i) + (1 + i) \Rightarrow a = -2$

Product of roots $= \dfrac{b}{1} = (1 - i)(1 + i) \Rightarrow b = 2$

5. **(b) :** Range $= 9 - 2 = 7$

6. **(a) :** We have,

$$\left|\frac{m - m_1}{1 + m_1 m}\right| = \left|\frac{m - m_2}{1 + m m_2}\right| \Rightarrow \left|\frac{m - 3}{1 + 3m}\right| = \left|\frac{(2m - 1)}{m + 2}\right|$$

$\Rightarrow$ $(m - 3)(m + 2) \pm (2m - 1)(1 + 3m) = 0$

$\Rightarrow$ $m^2 + 1 = 0$ or $7m^2 - 2m - 7 = 0 \Rightarrow m = \dfrac{1 \pm 5\sqrt{2}}{7}$

7. **(a) :** $n(A) = 5 \Rightarrow n(P(A)) = 2^5 = 32$

8. **(b) :** If $n(A) = m$ and $n(B) = n$, then total number of relations from A to B is 2^{mn}.

9. **(b) :** $\lim\limits_{x \to -1} [1 + x + x^2 + \ldots + x^{10}]$

$= 1 + (-1) + (-1)^2 + \ldots + (-1)^{10} = 1 - 1 + 1 - \ldots + 1 = 1$

10. **(b) :** Mean $(\bar{x}) = \dfrac{2 + 9 + 9 + 3 + 6 + 9 + 4}{7} = \dfrac{42}{7} = 6$

M.D. $(\bar{x}) = \dfrac{\Sigma|x_i - \bar{x}|}{n} = \dfrac{4 + 3 + 3 + 3 + 0 + 3 + 2}{7} = 2.57$

11. **(b) :** We have, $x = 1 + 2i$

$\Rightarrow$ $x - 1 = 2i \Rightarrow x^2 - 2x + 5 = 0$

Now, $x^3 + 7x^2 - x + 16$

$= x(x^2 - 2x + 5) + 9(x^2 - 2x + 5) + (12x - 29)$

$= x(0) + 9(0) + 12x - 29 = 12(1 + 2i) - 29 = -17 + 24i$

12. **(d) :** Let m be the slope of the line, which is perpendicular to the line $x + y + 7 = 0$.

The slope of the given line $y = (-1)x - 7$ is -1. Therefore, using the condition of perpendicularity of lines, we have $m \times (-1) = -1$ or $m = 1$

Hence, the required equation of the line is

$y - 2 = (1)(x - 1) \Rightarrow y - 2 = x - 1 \Rightarrow x - y + 1 = 0$

13. **(a) :** We have, $x^2 + 2 = 0 \Rightarrow x^2 = -2 \Rightarrow x = \pm\sqrt{-2} = \pm\sqrt{2}i$

14. **(b) :** Equation of line in intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{a} + \frac{y}{b} - 1 = 0$$

Its distance from origin is

$$\left|\frac{0 + 0 - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}\right| = p \Rightarrow p = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \Rightarrow \frac{1}{p} = \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$$

On squaring both sides, we get $\dfrac{1}{p^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2}$

15. **(a) :** Subset of $\{\ \}$ *i.e.* ϕ is ϕ.

Subsets of $\{4\}$ are ϕ, $\{4\}$.

Subsets of $\{4, 5\}$ are $\phi, \{4\}, \{5\}, \{4, 5\}$.

Subsets of $\{0\}$ are $\phi, \{0\}$.

16. **(d) :** Given, $f(x) = \dfrac{x}{x^2 + 1} \Rightarrow f(2) = \dfrac{2}{2^2 + 1} = \dfrac{2}{5}$

$$\Rightarrow f(f(2)) = f\left(\frac{2}{5}\right) = \frac{\frac{2}{5}}{\left(\frac{2}{5}\right)^2 + 1} = \frac{\frac{2}{5}}{\frac{4}{25} + 1} = \frac{\frac{2}{5}}{\frac{29}{25}} = \frac{10}{29}$$

17. **(b) :** We have, $\lim\limits_{x \to 2a} \dfrac{\sqrt{x - 2a} + \sqrt{x} - \sqrt{2a}}{\sqrt{x^2 - 4a^2}}$

$$= \lim_{x \to 2a} \frac{\sqrt{x - 2a} + \dfrac{x - 2a}{\sqrt{x} + \sqrt{2a}}}{\sqrt{x - 2a}\sqrt{x + 2a}}$$

$$= \lim_{x \to 2a}\left(\frac{1}{\sqrt{x + 2a}} + \frac{\sqrt{x - 2a}}{(\sqrt{x} + \sqrt{2a})\sqrt{x + 2a}}\right) = \frac{1}{2\sqrt{a}} + 0 = \frac{1}{2\sqrt{a}}$$

18. **(c) :** Given equation of line is $3x - 4y - 26 = 0$.

Here, $A = 3, B = -4, C = -26$ and the point $(x_1, y_1) = (3, -5)$.

The distance of the given point from given line is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \frac{|3 \cdot 3 + (-4)(-5) - 26|}{\sqrt{3^2 + (-4)^2}} = \frac{3}{5} \text{ unit}$$

19. **(d) :** Variance $= \sigma^2$

$$\Rightarrow \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\Sigma x_i\right)^2 = \sigma^2$$

$\Rightarrow \quad 0 = \frac{1}{n}(400) - (5)^2$

$\Rightarrow \quad \frac{400}{n} = 25 \;\Rightarrow\; n = 16$

20. (b) : Variance of 1, 2, 3, 4, 5, …, 10 is $\frac{99}{12}$.

$\therefore$ Variance of 3, 6, 9, 12, … , 30 is $(3^2) \times \frac{99}{12} = 9 \times \frac{99}{12}$

$\therefore$ S.D. of 3, 6, 9, 12, ..., 30 $= \sqrt{9 \times \frac{99}{12}} = \frac{3}{2}\sqrt{33}$

21. (c) : $\lim\limits_{x\to 0}\frac{\sin 3x}{\sin 2x} = \lim\limits_{x\to 0}\left(\frac{\sin 3x}{3x} \times \frac{2x}{\sin 2x} \times \frac{3}{2}\right)$

$= \frac{3}{2} \cdot \lim\limits_{3x\to 0}\frac{\sin 3x}{3x} \times \lim\limits_{2x\to 0}\frac{2x}{\sin 2x}$

$(\because$ as $x \to 0,\ 3x \to 0$ and $2x \to 0)$

$= \frac{3}{2} \times 1 \times 1 = \frac{3}{2}$

22. (c) : $|x - 3| < 2 \Rightarrow 3 - 2 < x < 3 + 2 \;\Rightarrow 1 < x < 5$

$\Rightarrow \quad x = 2, 3, 4$

$\therefore$ Required set = {2, 3, 4}

23. (b) : $f(x) = x^2 - \frac{1}{x^2} \Rightarrow f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^2 - \frac{1}{(1/x)^2}$

$= \frac{1}{x^2} - x^2 = -\left(x^2 - \frac{1}{x^2}\right) = -f(x)$

24. (c) : We have, $\lim\limits_{x\to\frac{\pi}{2}}\frac{1-\cos x}{\sin x} = \lim\limits_{x\to\frac{\pi}{2}}\frac{2\sin^2\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}$

$= \lim\limits_{x\to\frac{\pi}{2}}\tan\left(\frac{x}{2}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

25. (d) : We have, $(-\sqrt{3}+\sqrt{-2})(2\sqrt{3}-i)$

$= (-\sqrt{3}+\sqrt{2}i)(2\sqrt{3}-i)$

$= -6 + \sqrt{3}i + 2\sqrt{6}i - \sqrt{2}i^2 = (-6+\sqrt{2}) + \sqrt{3}(1+2\sqrt{2})i$

26. (c) : Here, $b^2 - 4ac = 1^2 - 4 \times 1 \times 1 = 1 - 4 = -3$

Therefore, the solutions are

$x = \frac{-1 \pm \sqrt{-3}}{2 \times 1} = \frac{-1 \pm \sqrt{3}i}{2}$

27. (c) : The mean $(\overline{x})$ of the given data is,

$\overline{x} = \frac{1}{20}\sum_{i=1}^{20} x_i = \frac{200}{20} = 10$

The absolute values of the deviations from mean, *i.e.*, $|x_i - x|$ are 2, 7, 8, 7, 6, 1, 7, 9, 10, 5, 2, 7, 8, 7, 6, 1, 7, 9, 10, 5

$\therefore \quad \sum_{i=1}^{20}|x_i - \overline{x}| = 124$ and M.D. $(\overline{x}) = \frac{124}{20} = 6.2$

28. (b) : Clearly, the required equation of line is of the form $2x - 3y + k = 0$, where k is an arbitrary constant. If it passes through (1, 1), then $2 - 3 + k = 0$

$\Rightarrow \quad k = 1$

$\therefore$ Required equation of line is $2x - 3y + 1 = 0$

29. (c) : Range = Maximum value – least value

$\Rightarrow \quad 148 = x - 96$

$\Rightarrow \quad x = 148 + 96 = 244$

30. (c) : Set of natural numbers N, set of rational numbers Q and set of concentric circles are infinite sets and there are 7 days in a week, which will form a finite set.

31. (b) : Since $x^2 - 5x + 4 = (x - 4)(x - 1)$, then the function f is defined for all real numbers except $x = 4$ and $x = 1$. Hence the domain of f is $R - \{1, 4\}$.

32. (c) : Here, $b^2 - 4ac = 1^2 - 4 \times \sqrt{5} \times \sqrt{5} = 1 - 20 = -19$

Therefore, the solutions are

$x = \frac{-1 \pm \sqrt{-19}}{2\sqrt{5}} = \frac{-1 \pm \sqrt{19}i}{2\sqrt{5}}$

33. (b) : Mid point of given line segment AB is (2, – 1). Also, slope of the line segment AB is – 8/8 = –1

$\therefore$ Slope of the required line is 1.

So, equation of the required line is

$y + 1 = 1(x - 2) \Rightarrow x - y = 3$

34. (c) : Given, $\lim\limits_{x\to\infty}\left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$

$\Rightarrow \quad \lim\limits_{x\to\infty}\left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x\left(\frac{a/x+b/x^2}{a/x+b/x^2}\right)} = e^2$

$\Rightarrow \quad e^{\lim\limits_{x\to\infty}\left(\frac{a}{x}+\frac{b}{x^2}\right)2x} = e^2 \qquad \left[\because \lim\limits_{x\to 0}(1+x)^{1/x} = e\right]$

$\Rightarrow \quad e^{\lim\limits_{x\to\infty}\left(a+\frac{b}{x}\right)2} = e^2$

$\therefore \quad 2a = 2 \;\Rightarrow\; a = 1$

$\therefore \quad a = 1$ and b can take any real value.

35. (c) : Since complex roots always exist in conjugate pair, so other root will be $2 - 3i$.

Now, S = sum of roots = $2 + 3i + 2 - 3i = 4$

and P = product of roots = $(2 + 3i)(2 - 3i) = 4 + 9 = 13$

So, required equation will be

$x^2 - Sx + P = 0$

$\Rightarrow \quad x^2 - 4x + 13 = 0$

36. (c) : $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 12 + 8 - 4 = 16$

37. (a) : We have $\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{\sin^3 x}$

$$=\lim_{x\to 0}\frac{\sin x\left(\frac{1}{\cos x}-1\right)}{\sin^3 x}=\lim_{x\to 0}\frac{1-\cos x}{\cos x\sin^2 x}$$

$$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{\cos x\left(4\sin^2\frac{x}{2}\cdot\cos^2\frac{x}{2}\right)}=\frac{1}{2}$$

38. (b) : Given, $(x_1, y_1) = (3, -1)$ and $(x_2, y_2) = (4, -2)$

Slope of line, $m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{-2+1}{4-3}=-1$

$\Rightarrow$ $\tan\theta = -1 \Rightarrow \tan\theta = -\tan 45°$

$\Rightarrow$ $\tan\theta = \tan(180° - 45°) \Rightarrow \tan\theta = \tan 135°$

$\Rightarrow$ $\theta = 135°$

39. (b) : $\arg\left(\dfrac{3+i}{2-i}+\dfrac{3-i}{2+i}\right)=\arg\left(\dfrac{6+5i+i^2+6-5i+i^2}{5}\right)$

$\Rightarrow \arg\left(\dfrac{10}{5}\right)=\arg(2)=0$

40. (a) : Arranging the given data in ascending order, we get 2, 3, 5, 7, 8, 9, 10, 11, 14. Here $n = 9$, which is odd.

$\therefore$ Median $=\dfrac{1}{2}(n+1)^{\text{th}}$ observation

$=\dfrac{1}{2}(9+1)^{\text{th}}$ observation $=5^{\text{th}}$ observation $=8$

Thus, $M = 8$

The values of $(x_i - M)$ are: $-6, -5, -3, -1, 0, 1, 2, 3, 6$.

$$\therefore\quad \sum_{i=1}^{9}|x_i-M|=(6+5+3+1+0+1+2+3+6)=27$$

$$\Rightarrow\ \text{M.D. }(M)=\frac{\sum\limits_{i=1}^{9}|x_i-M|}{9}=\frac{27}{9}=3$$

Hence, M.D. $(M) = 3$.

41. (b) : $n(A\cup B) = n(A) + n(B) - n(A\cap B)$
$= 300 + 400 - 100 = 600$

42. (b) : $f(x) = g(x) \Rightarrow 2x^2 - 1 = 1 - 3x$

$\Rightarrow$ $2x^2 + 3x - 2 = 0 \Rightarrow 2x^2 + 4x - x - 2 = 0$

$\Rightarrow$ $(2x - 1)(x + 2) = 0$

Thus domain for which the function $f(x) = g(x)$ is $\left\{-2, \dfrac{1}{2}\right\}$.

43. (b) : The first $2n$ terms will cancel in pairs and the last term is $(i^2)^{2n+1} = i^{4n}\cdot i^2 = -1$

44. (a) : Slope of the line $=\dfrac{-2}{5}$

$\Rightarrow\ \dfrac{a-4}{-2-3}=\dfrac{-2}{5}\Rightarrow\dfrac{a-4}{-5}=\dfrac{-2}{5}\ \Rightarrow a-4=2\Rightarrow a=6$

45. (c) : Standard deviation of ungrouped data is given by

$$\sigma=\sqrt{\frac{\sum(x_i-\bar{x})^2}{n}}$$

Here, $\bar{x}=\dfrac{12+23+34+45+56+67+78}{7}=45$

Now, $\sum\limits_{i=1}^{7}(x_i-\bar{x})^2=(12-45)^2+(23-45)^2+(34-45)^2$

$+ (45 - 45)^2 + (56 - 45)^2 + (67 - 45)^2 + (78 - 45)^2$

$= 1089 + 484 + 121 + 0 + 121 + 484 + 1089 = 3388$

$\therefore\quad \sigma=\sqrt{\dfrac{3388}{7}}=\sqrt{484}=22.$

46. (b) : We have, a_2 = ₹ 13750

$\Rightarrow$ $a + d = 13750$ ...(i)

and a_5 = ₹ 17500

$\Rightarrow$ $a + 4d = 17500$ ...(ii)

Solving (i) and (ii), we get

$3d = 3750$

$\Rightarrow$ d = ₹ 1250

$\therefore$ Fixed amount by which salary is increasing every year is ₹ 1250.

47. (a) : From (i) $a + 1250 = 13750$

$\Rightarrow$ a = ₹ 12500

Thus, initial salary of Jashanpreet is ₹ 12500

48. (c) : Monthly salary of Jashanpreet in 10th year is given by $a_{10} = a + 9d = 12500 + (9 \times 1250)$ = ₹ 23750

49. (d) : Total (salary) amount received by Jashanpreet in 10 years

$$=12\times\left[\frac{10}{2}[12500+23750]\qquad\left[\because S_n=\frac{n}{2}(a+l)\right]\right]$$

$=12\times\left[5\,[36250]\,\right]$ = 12 × 181250 = ₹ 2175000

50. (c) : $S_1=\dfrac{n}{2}[2a+(n-1)d]$; $S_2=\dfrac{2n}{2}[2a+(2n-1)d]$

$\therefore\quad S_2-S_1=na+(3n-1)\dfrac{nd}{2}=\dfrac{n}{2}[2a+(3n-1)d]$

Also, $S_3=\dfrac{3n}{2}[2a+(3n-1)d]=3(S_2-S_1)$

$\therefore\quad \dfrac{S_3}{S_2-S_1}=3$

◆ ◆

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1.	(b)	2.	(c)	3.	(b)	4.	(a)	5.	(b)
6.	(c)	7.	(c,d)	8.	(b,d)	9.	(c,d)	10.	(b,d)
11.	(a,d)	12.	(b,c)	13.	(d)	14.	(d)	15.	(b)
16.	(c)	17.	(4)	18.	(2)	19.	(3)	20.	(9)

MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 6 : Sequences and Series

Time Taken : 60 Min.

Only One Option Correct Type

1. The ratio of sum of first three terms of a G.P. to the sum of first six terms is 64 : 91, the common ratio of G.P. is

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{5}{4}$ (d) $\frac{7}{4}$

2. If $\frac{b+c-2a}{a}, \frac{c+a-2b}{b}, \frac{a+b-2c}{c}$ are in A.P., then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in

(a) G.P. (b) H.P.
(c) A.P. (d) None of these

3. If $a^2 + 4b^2 + 9c^2 = 2ab + 6bc + 3ac$, where a, b, c are non zero numbers, then a, b, c are in

(a) G.P. (b) H.P.
(c) A.P. (d) None of these

4. The sum of n terms of the series

$\frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \frac{1}{7\cdot 9} + \ldots$ is

(a) $\frac{n}{3(2n+3)}$ (b) $\frac{n+1}{2n+3}$
(c) $\frac{n-1}{2n+1}$ (d) $\frac{n}{2n+1}$

5. If the m^{th}, n^{th} and p^{th} terms of A.P. & G.P. be equal & are respectively x, y, z, then which of the following hold good?

(a) $xyz = 1$ (b) $x^y y^z z^x = x^z y^x z^y$
(c) $x^y y^z z^x = 1$ (d) $x^z y^x z^y = 1$

6. If $x > 1, y > 1, z > 1$ are in G.P., then $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$ are in

(a) A.P. (b) G.P.
(c) H.P. (d) none of these

One or More Than One Option(s) Correct Type

7. The value of $\sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{j} 1$ is equal to

(a) Σn^2 (b) Σn^3

(c) $\frac{n(n+1)(n+2)}{3!}$

(d) Coefficient of x^3 in the expansion of $(1 + x)^{n+2}$

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8. There are two numbers a and b whose product is 192 and the quotient of A.M. by H.M. of their greatest common divisor and least common multiple is $\frac{169}{48}$. The smaller of a and b is

(a) 2 (b) 4 (c) 6 (d) 12

9. If $a_1, a_2, \ldots, a_{50}$ are in G.P., then the value of $\frac{a_1 - a_3 + a_5 - \ldots + a_{49}}{a_2 - a_4 + a_6 - \ldots + a_{50}}$ is equal to

(a) 0 (b) 1

(c) $\frac{a_1}{a_2}$ (d) $\frac{a_{24}}{a_{25}}$

10. Consider an A.P. $a_1, a_2, a_3, \ldots, a_n$ and the G.P. $b_1, b_2, b_3, \ldots, b_n$ such that $a_1 = b_1 = 1$,

$a_9 = b_9$ and $\sum_{r=1}^{9} a_r = 369$, then

(a) $b_6 = 27$ (b) $b_7 = 27$

(c) $b_8 = 81$ (d) $b_9 = 81$

11. If $5x - y$, $2x + y$, $x + 2y$ are in A.P. and $(x - 1)^2$, $(xy + 1)$, $(y + 1)^2$ are in G.P., $x \neq 0$, then $x + y =$

(a) $\frac{3}{4}$ (b) 3 (c) -5 (d) -6

12. The integer part of the sum $\sum_{n=1}^{16} \frac{n^4}{4n^2 - 1}$ is a multiple of

(a) 2 (b) 3 (c) 5 (d) 7

Comprehension Type

Paragraph for Q. No. 13 to 15

Let a_n and b_n be n^{th} terms of two arithmetic progressions where $n \in N$ and their sum of first n-terms be $S_a(n)$ and $S_b(n)$ respectively. Given $\frac{S_a(n)}{S_b(n)} = \frac{5n+9}{2n+8}$ and $a_2 = 4$.

13. The value of $\frac{a_1}{b_1}$ is

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$

(c) $\frac{6}{5}$ (d) $\frac{7}{5}$

14. Common difference of the sequence a_n is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

15. The value of b_4 is

(a) $\frac{8}{3}$ (b) $\frac{11}{3}$

(c) 8 (d) 11

Matrix Match Type

16. Match the following:

	Column-I		Column-II (Sum of the digits of S)
P.	$S = 1^2 - 2^2 + 3^2 + \ldots$ to 21 terms	1.	4
Q.	$S = 1^3 - 2^3 + 3^3 - \ldots$ to 15 terms	2.	14
R.	$S = 1^2 + 3^2 + 5^2 + \ldots$ to 8 terms	3.	18
S.	$S = 1^3 + 3^3 + 5^3 + \ldots$ to 6 terms	4.	20

	P	Q	R	S
(a)	1	4	3	2
(b)	2	3	1	4
(c)	1	4	2	3
(d)	2	3	4	1

Numerical Answer Type

17. Consider the sequence of integers 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, ... where n occurs n times. If n occurs as 2017th term, then the last digit of n is __________.

18. A G.P. consists of even number of terms. If the sum of all terms is three times that of all odd terms, then the common ratio is __________.

19. Let N be the number of ordered pairs (a, b) of natural numbers such that $a < b$ and the H.M. of a and b is 2010. The last digit of N is __________.

20. The length, breadth and height of a rectangular box are in G.P. The volume is 27. If the total surface area is 78, then the length is __________.

* * *

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SELF CHECK

No. of questions attempted ......

No. of questions correct ......

Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
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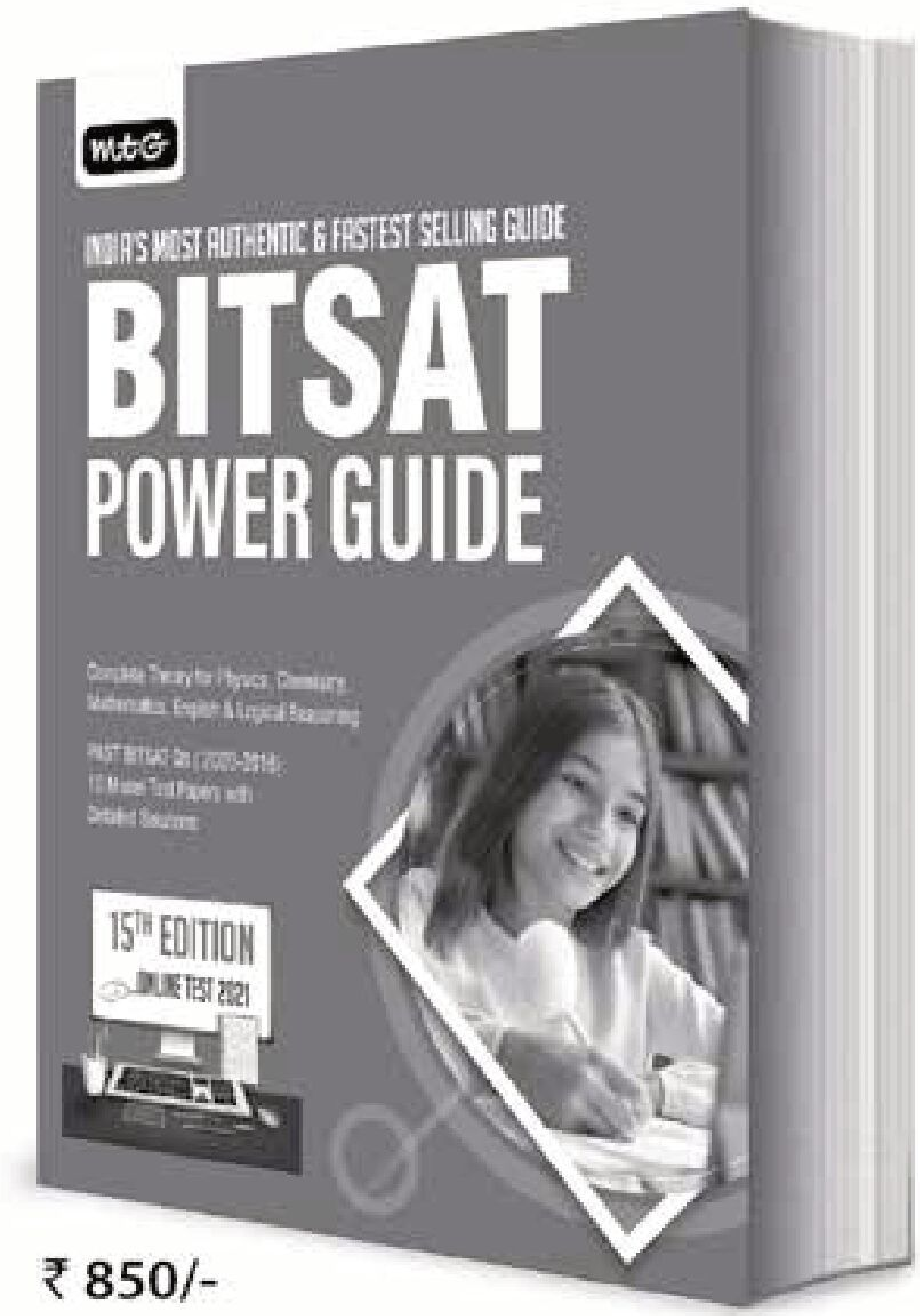

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CLASS-XII

TERM-I OBJECTIVE TYPE QUESTIONS

Practice Paper 2021

Time allowed : 90 minutes
Maximum marks : 40

General Instructions :

1. *This question paper contains three sections A, B and C. Each part is compulsory.*
2. *Section - A has 20 MCQs, attempt any 16 out of 20.*
3. *Section - B has 20 MCQs, attempt any 16 out of 20*
4. *Section - C has 10 MCQs, attempt any 8 out of 10.*
5. *There is no negative marking.*
6. *All questions carry equal marks.*

Mark your answer in the Sample OMR Sheet given after Practice Paper to get familiar with the actual paper pattern.

SECTION - A

In this section, attempt any 16 questions out of Questions 1-20. Each Question is of 1 mark weightage.

1. Evaluate : $\cot\left(\frac{\pi}{2} - 2\cot^{-1}\sqrt{3}\right)$

(a) $-\sqrt{3}$ (b) $\sqrt{3}$ (c) $\sqrt{2}$ (d) $-\sqrt{2}$

2. Find the value of k so that the following function is continuous at $x = 2$.

$$f(x) = \begin{cases} \dfrac{x^3 + x^2 - 16x + 20}{(x-2)^2}; & x \neq 2 \\ k; & x = 2 \end{cases}$$

(a) 5 (b) 6 (c) 7 (d) 8

3. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, then find $(x - y)$.

(a) 10 (b) 12 (c) 8 (d) 14

4. If $A = \begin{bmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{bmatrix}$ is a symmetric matrix, then $x =$

(a) 4 (b) 3 (c) –4 (d) –3

5. The function $f(x) = \cot^{-1}x + x$ increases in the interval

(a) $(1, \infty)$ (b) $(-1, \infty)$ (c) $(0, \infty)$ (d) $(-\infty, \infty)$

6. Find the co-factors of elements a_{12}, a_{22}, a_{32}, respectively, of the matrix $\begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$.

(a) $0, 2, -2\sin\theta$ (b) $2, 0, 2\sin\theta$
(c) $2, 0, -2\sin\theta$ (d) $-2\sin\theta, 2, 0$

7. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha R\beta \Leftrightarrow \alpha \perp \beta$, $\alpha, \beta \in L$. Then, R is

(a) Reflexive only (b) Symmetric only
(c) Transitive only (d) None of these

8. If $\begin{bmatrix} x \cdot y & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, then find the value of $(x + y + z)$.

(a) 0 (b) 1 (c) 2 (d) 3

9. Tangents to the curve $y = x^3 + 3x$ at $x = -1$ and $x = 1$ are
(a) parallel
(b) intersecting obliquely but not at an angle of 45°
(c) intersecting at right angles
(d) intersecting at an angle of 45°

10. $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) + 4\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ is equal to
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{4\pi}{3}$ (d) $\frac{3\pi}{4}$

11. Let $g(x) = x^2 - 4x - 5$, then
(a) $g(x)$ is one-one on R
(b) $g(x)$ is not one-one on R
(c) $g(x)$ is bijective on R
(d) None of these

12. If $y = \log_e\left(\frac{x^2}{e^2}\right)$, then $\frac{d^2y}{dx^2}$ equals
(a) $-\frac{1}{x}$ (b) $-\frac{1}{x^2}$ (c) $\frac{2}{x^2}$ (d) $-\frac{2}{x^2}$

13. If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and $A = A^T$, then
(a) $x = 0, y = 5$ (b) $x + y = 5$
(c) $x = y$ (d) $x = 0, y = 9$

14. If $x = \frac{2at}{1+t^3}$ and $y = \frac{2at^2}{(1+t^3)^2}$, then $\frac{dy}{dx}$ is
(a) ax (b) a^2x^2 (c) $\frac{x}{a}$ (d) $\frac{x}{2a}$

15. If $A = \begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$, then find the co-factor of the element a_{21} of its 2nd row.
(a) 3 (b) -3 (c) 2 (d) -2

16. The equation of the normal to the curve $y^2 = 8x$ at the origin is
(a) $2y - x = 0$ (b) $x - y = 0$
(c) $x - 2y = 0$ (d) $y = 0$

17. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then the value of $|\text{adj } A|$ is
(a) 64 (b) 16 (c) 0 (d) -8

18. If $\log_{10}\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = 2$, then $\frac{dy}{dx} =$
(a) $-\frac{99x}{101y}$ (b) $\frac{99x}{101y}$
(c) $-\frac{99y}{101x}$ (d) $\frac{99y}{101x}$

19. The maximum value of $Z = 10x + 25y$ subject to $0 \le x \le 3$, $0 \le y \le 3$, $x + y \le 5$ is
(a) 75 (b) 80 (c) 95 (d) 105

20. The function $f(x) = \frac{x}{\log x}$ increases in the interval
(a) $(0, \infty)$ (b) $(0, e)$
(c) (e, ∞) (d) $(-\infty, 0)$

SECTION - B

In this section, attempt any 16 questions out of the Questions 21 - 40. Each Question is of 1 mark weightage.

21. The domain of the function $\cos^{-1}(\log_2(x^2 + 5x + 8))$ is
(a) $[2, 3]$ (b) $[-2, 2]$ (c) $[3, 1]$ (d) $[-3, -2]$

22. If $\log y = m\tan^{-1} x$, then
(a) $(1 + x^2)y_2 + (2x + m)y_1 = 0$
(b) $(1 + x^2)y_2 + (2x - m)y_1 = 0$
(c) $(1 + x^2)y_2 - (2x + m)y_1 = 0$
(d) $(1 + x^2)y_2 - (2x - m)y_1 = 0$

23. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20), is
(a) $p = q$ (b) $p = 2q$ (c) $q = 2p$ (d) $q = 3p$

24. If $y = a^{x^{a^{x^{a^{x \ldots \infty}}}}}$, then the value of $\frac{dy}{dx}$ is
(a) $\frac{2}{x(1 - y\log x)}$ (b) $\frac{y^2 \log y}{x(1 - y\log x)}$
(c) $\frac{y^2 \log y}{x(1 - y\log x \log y)}$ (d) $\frac{y^2 \log y}{x(1 + y\log x \log y)}$

25. If the area of a triangle ABC, with vertices $A(1, 3)$, $B(0, 0)$ and $C(k, 0)$ is 3 sq. units, then the value of k is
(a) 2 (b) 3 (c) 4 (d) 5

26. The slope of normal to the curve $y = x^3 + 2x + 6$ which is parallel to line $x + 14y + 4 = 0$, is
(a) $-\frac{1}{3}$ (b) $-\frac{1}{14}$ (c) -4 (d) $-\frac{1}{2}$

27. Using the principal values, the value of

$$\sin^{-1}\left\{\sin\frac{5\pi}{6}\right\}+\tan^{-1}\left\{\tan\frac{\pi}{6}\right\}=$$

(a) $\pi/6$ (b) $2\pi/3$ (c) $\pi/3$ (d) $5\pi/6$

28. Find the minor of 6 and co-factor of 4 respectively in the determinant, $\Delta=\begin{vmatrix}1&2&3\\4&5&6\\7&8&9\end{vmatrix}$.

(a) 6, 6 (b) 6, –6 (c) –6, –6 (d) –6, 6

29. If the function $f(x)=\dfrac{a\sin x+2\cos x}{\sin x+\cos x}$ is strictly increasing for all values of x, then

(a) $a<1$ (b) $a>1$ (c) $a<2$ (d) $a>2$

30. The function $f: R \to R$ given by $f(x)=x^2$, $x \in R$, is

(a) one - one but not onto

(b) onto but not one - one

(c) both one - one and onto

(d) neither one - one nor onto

31. The value(s) of x for which the function

$$f(x)=\begin{cases}1-x & ,\ x<1\\(1-x)(2-x) & ,1\le x\le 2\\3-x & ,\ x>2\end{cases}$$ fails to be continuous is (are)

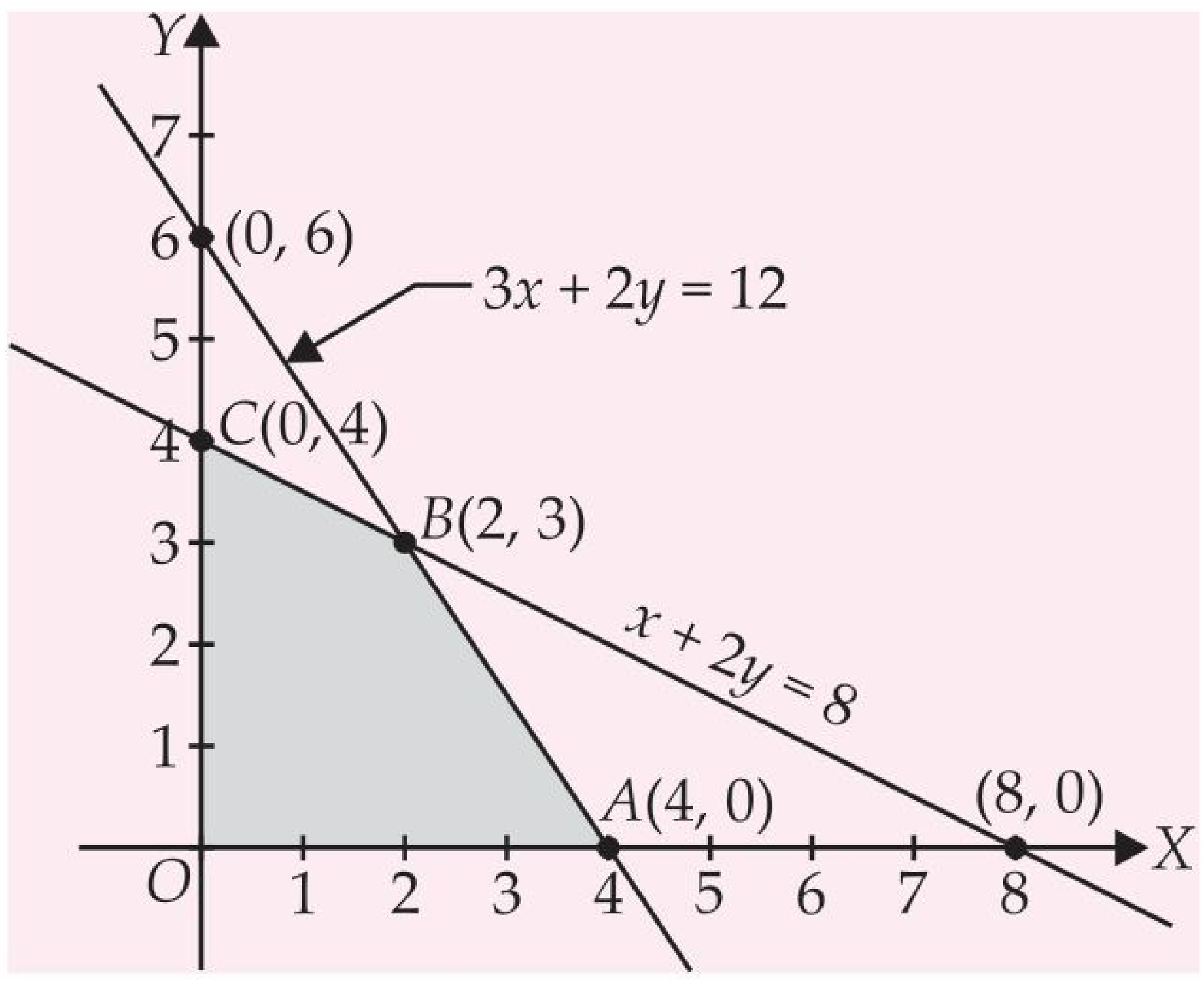
(a) 1 (b) 2

(c) 3 (d) all real numbers

32. If $A=\begin{bmatrix}2x&0\\x&x\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}1&0\\-1&2\end{bmatrix}$, then x equals

(a) 2 (b) $-\dfrac{1}{2}$ (c) 1 (d) $\dfrac{1}{2}$

33. In the given graph, the feasible region for a LPP is shaded. Find the minimum value of the objective function $Z=-3x+4y$.



(a) –6 (b) –12 (c) 6 (d) 16

34. It is given that at $x=1$, the function x^4-62x^2+ax+9 attains its maximum value on the interval [0, 2]. Find the value of a.

(a) 100 (b) 120 (c) 140 (d) 160

35. If $A=\begin{bmatrix}0&2&-3\\-2&0&-1\\3&1&0\end{bmatrix}$, then A is a

(a) symmetric matrix

(b) skew-symmetric matrix

(c) diagonal matrix (d) unit matrix

36. $\sin(\cot^{-1}x)=$

(a) $\dfrac{1}{\sqrt{2+x^2}}$ (b) $\dfrac{-1}{\sqrt{1+x^2}}$

(c) $\dfrac{1}{\sqrt{1-x^2}}$ (d) $\dfrac{1}{\sqrt{1+x^2}}$

37. Let R be the relation on the set **R** of all real numbers defined by aRb iff $|a-b|\le 1$. Then R is

(a) reflexive only

(b) symmetric only

(c) both reflexive and symmetric

(d) none of these

38. If $A=\begin{bmatrix}\cos\alpha&-\sin\alpha&0\\\sin\alpha&\cos\alpha&0\\0&0&1\end{bmatrix}$, then find adj A.

(a) $\begin{bmatrix}\cos\alpha&\sin\alpha&0\\-\sin\alpha&\cos\alpha&0\\0&0&1\end{bmatrix}$ (b) $\begin{bmatrix}\cos\alpha&-\sin\alpha&0\\-\sin\alpha&\cos\alpha&0\\0&0&1\end{bmatrix}$

(c) $\begin{bmatrix}\cos\alpha&\sin\alpha&0\\\sin\alpha&\cos\alpha&0\\0&0&1\end{bmatrix}$ (d) $\begin{bmatrix}-\cos\alpha&-\sin\alpha&0\\\sin\alpha&\cos\alpha&0\\0&0&1\end{bmatrix}$

39. Slope of normal to the curve $y=x^2-\dfrac{1}{x^2}$ at $(-1, 0)$ is

(a) $-\dfrac{1}{4}$ (b) –4 (c) $\dfrac{1}{4}$ (d) 4

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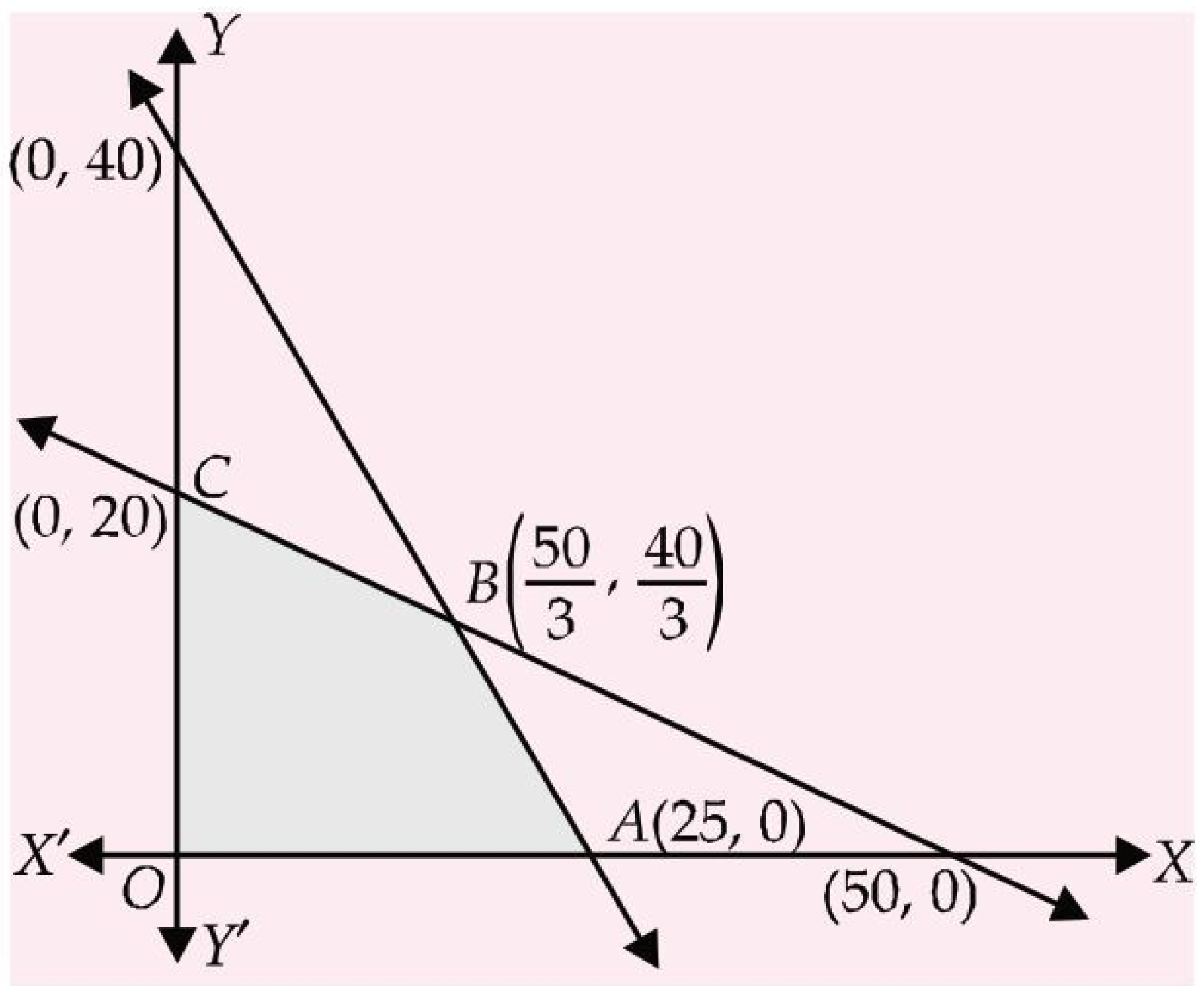
40. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

(a) $\alpha = a^2 + b^2, \beta = ab$
(b) $\alpha = a^2 + b^2, \beta = 2ab$
(c) $\alpha = a^2 + b^2, \beta = a^2 - b^2$
(d) $\alpha = 2ab, \beta = a^2 + b^2$

SECTION - C

- *In this section, attempt any 8 questions.*
- *Each question is of 1-mark weightage.*
- *Questions 46-50 are based on a Case-Study.*

41. The feasible region for a LPP is shown in the figure. Let $Z = x + y$ be the objective function, then the maximum value of Z occurs at



(a) a unique point (b) no point
(c) infinitely many points
(d) two points only

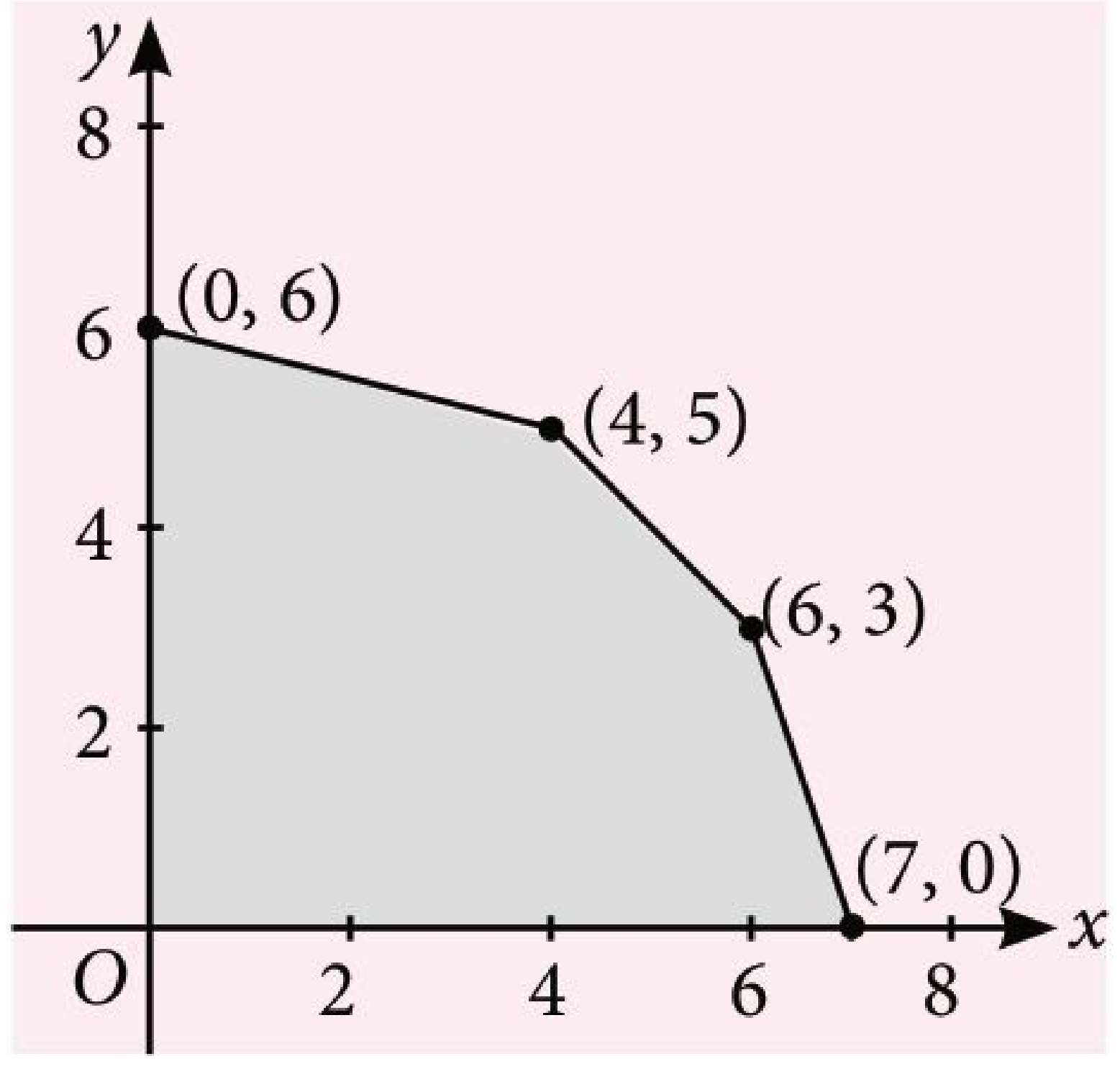
42. The slope of the tangent to the curve $y^2e^{xy} = 9e^{-3}x^2$ at $(-1, 3)$ is

(a) $\frac{-15}{2}$ (b) $\frac{-9}{2}$ (c) 15 (d) $\frac{15}{2}$

43. If $y = x^3 + x^2 + x + 1$, then y

(a) has a local minimum
(b) has a local maximum
(c) neither have a local minimum nor local maximum
(d) None of these

44. The feasible region for an LPP is shown in the figure. Let $Z = 2x + 5y$ be the objective function. Maximum of Z occurs at

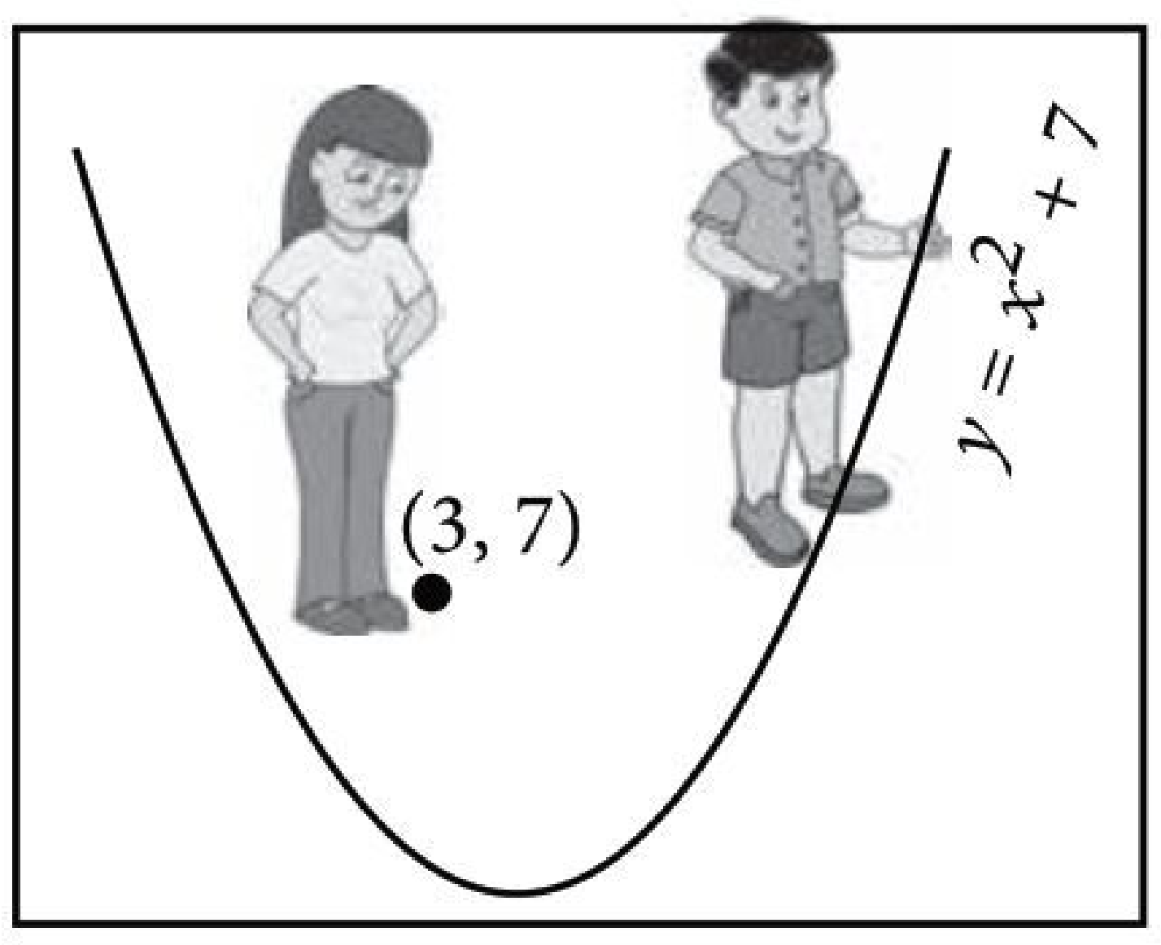


(a) (7, 0) (b) (6, 3)
(c) (0, 6) (d) (4, 5)

45. Find x, if $A = \begin{bmatrix} 1 & 2 & x \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$ is singular.

(a) 1 (b) 2 (c) 3 (d) 4

CASE STUDY



A student Arun is running in a playground along the curve given by $y = x^2 + 7$. Another student Manita standing at point (3, 7) in playground wants to hit Arun by paper ball when Arun is nearest to Manita.
"Assume 'D' is the distance between Arun and Manita."
Based on the above information, answer the following questions.

46. Arun's position on the curve will be given by

(a) $(x^2, y - 7)$ (b) $(x^2, y + 7)$
(c) $(x, x^2 + 7)$ (d) $(x^2, x - 7)$

47. Distance (D) is given by

(a) $(x - 1)(2x^2 + 2x + 3)$
(b) $(x - 3)^2 + x^4$
(c) $\sqrt{(x-3)^2 + x^4}$
(d) $\sqrt{(x-1)(2x^2 + 2x + 3)}$

48. For which real value(s) of x, first derivative of D^2 w.r.t. 'x' will Vanish?

(a) 1 (b) 2 (c) 3 (d) 4

49. Find the position of Arun when Manita will hit him with the paper ball.

(a) (5, 32) (b) (1, 8) (c) (3, 7) (d) (3, 16)

50. The minimum value of D is

(a) 3 (b) $\sqrt{3}$ (c) 5 (d) $\sqrt{5}$

SAMPLE OMR SHEET

General Instructions

- *Details of the candidates will be pre-filled in the OMR.*
- *Candidate has to write a Question Paper Code in the upper right corner in the space provided for the same. Question Paper Code will be written on the Question Paper.*
- *Candidate has to write in his/her own hand in the space provided **"I confirm that all particulars given above are correct"** and sign in the running hand on OMR.*
- *Only Blue/Black ball point pen is to be used for filling desired information and for giving responses.*
- *Use of pencil is strictly prohibited.*

Important features of the OMR:

- *There is a space for answering 60 questions in an OMR irrespective of total questions given in the Question Papers of any subject.*
- *Answers given after the maximum question number in your question paper will not be evaluated.*

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Senior Secondary School Examination (Class XII)

अनुक्रमांक **Roll No.**	
(अंकों में एवं शब्दों में) (in digits and words)	
परीक्षा का दिन एवं तिथि Day & Date of the Examination	Monday & 06-12-2021
केन्द्र संख्या व नाम Center No. & Name	
विषय कोड व नाम Subject code & Name	041 & Mathematics
अभ्यार्थी का नाम Candidate's Name	
पिता का नाम Father's Name	
विद्यालय का कोड व नाम School code and Name	

OMR No.

प्रश्न पत्र के ऊपर लिखे कोड को दर्शाएं
Write code No. written on the question paper
Question Paper Code

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SOME EXAMPLES OF WRONG/CORRECT WAYS OF MARKING AS FOLLOWS

WRONG METHOD					CORRECT METHOD				
1	✓	⊗	ⓒ	ⓓ	1	●	ⓑ	ⓒ	ⓓ
2	ⓐ	✓	ⓒ	ⓓ	2	ⓐ	●	ⓒ	ⓓ
3	ⓐ	ⓑ	✓	●	3	ⓐ	ⓑ	●	ⓓ
4	ⓐ	ⓑ	ⓒ	◘	4	ⓐ	ⓑ	ⓒ	●
5	●	⊗	ⓒ	ⓓ	5	●	ⓑ	ⓒ	ⓓ
6	ⓐ	ⓑ	⊗	ⓓ	6	ⓐ	ⓑ	ⓒ	ⓓ
7	ⓐ	ⓑ	◒	ⓓ	7	ⓐ	ⓑ	●	ⓓ
8	ⓐ	⊖	ⓒ	●	8	ⓐ	ⓑ	ⓒ	●

प्रत्येक प्रश्न का उत्तर **केवल एक ही गोले में** गहरा निशान लगाकर दीजिए एवं उत्तर को बोक्स में लिखें Darken **ONLY ONE CIRCLE** for each Question and write the answer in the box.

Q.No.	Response	*	#	Q.No.	Response	*	#	Q.No.	Response	*	#	Q.No.	Response	*	#
1	ⓐ ⓑ ⓒ ⓓ		①	16	ⓐ ⓑ ⓒ ⓓ		⑯	31	ⓐ ⓑ ⓒ ⓓ		㉛	46	ⓐ ⓑ ⓒ ⓓ		㊻
2	ⓐ ⓑ ⓒ ⓓ		②	17	ⓐ ⓑ ⓒ ⓓ		⑰	32	ⓐ ⓑ ⓒ ⓓ		㉜	47	ⓐ ⓑ ⓒ ⓓ		㊼
3	ⓐ ⓑ ⓒ ⓓ		③	18	ⓐ ⓑ ⓒ ⓓ		⑱	33	ⓐ ⓑ ⓒ ⓓ		㉝	48	ⓐ ⓑ ⓒ ⓓ		㊽
4	ⓐ ⓑ ⓒ ⓓ		④	19	ⓐ ⓑ ⓒ ⓓ		⑲	34	ⓐ ⓑ ⓒ ⓓ		㉞	49	ⓐ ⓑ ⓒ ⓓ		㊾
5	ⓐ ⓑ ⓒ ⓓ		⑤	20	ⓐ ⓑ ⓒ ⓓ		⑳	35	ⓐ ⓑ ⓒ ⓓ		㉟	50	ⓐ ⓑ ⓒ ⓓ		㊿
6	ⓐ ⓑ ⓒ ⓓ		⑥	21	ⓐ ⓑ ⓒ ⓓ		㉑	36	ⓐ ⓑ ⓒ ⓓ		㊱	51	ⓐ ⓑ ⓒ ⓓ		(51)
7	ⓐ ⓑ ⓒ ⓓ		⑦	22	ⓐ ⓑ ⓒ ⓓ		㉒	37	ⓐ ⓑ ⓒ ⓓ		㊲	52	ⓐ ⓑ ⓒ ⓓ		(52)
8	ⓐ ⓑ ⓒ ⓓ		⑧	23	ⓐ ⓑ ⓒ ⓓ		㉓	38	ⓐ ⓑ ⓒ ⓓ		㊳	53	ⓐ ⓑ ⓒ ⓓ		(53)
9	ⓐ ⓑ ⓒ ⓓ		⑨	24	ⓐ ⓑ ⓒ ⓓ		㉔	39	ⓐ ⓑ ⓒ ⓓ		㊴	54	ⓐ ⓑ ⓒ ⓓ		(54)
10	ⓐ ⓑ ⓒ ⓓ		⑩	25	ⓐ ⓑ ⓒ ⓓ		㉕	40	ⓐ ⓑ ⓒ ⓓ		㊵	55	ⓐ ⓑ ⓒ ⓓ		(55)
11	ⓐ ⓑ ⓒ ⓓ		⑪	26	ⓐ ⓑ ⓒ ⓓ		㉖	41	ⓐ ⓑ ⓒ ⓓ		㊶	56	ⓐ ⓑ ⓒ ⓓ		(56)
12	ⓐ ⓑ ⓒ ⓓ		⑫	27	ⓐ ⓑ ⓒ ⓓ		㉗	42	ⓐ ⓑ ⓒ ⓓ		㊷	57	ⓐ ⓑ ⓒ ⓓ		(57)
13	ⓐ ⓑ ⓒ ⓓ		⑬	28	ⓐ ⓑ ⓒ ⓓ		㉘	43	ⓐ ⓑ ⓒ ⓓ		㊸	58	ⓐ ⓑ ⓒ ⓓ		(58)
14	ⓐ ⓑ ⓒ ⓓ		⑭	29	ⓐ ⓑ ⓒ ⓓ		㉙	44	ⓐ ⓑ ⓒ ⓓ		㊹	59	ⓐ ⓑ ⓒ ⓓ		(59)
15	ⓐ ⓑ ⓒ ⓓ		⑮	30	ⓐ ⓑ ⓒ ⓓ		㉚	45	ⓐ ⓑ ⓒ ⓓ		㊺	60	ⓐ ⓑ ⓒ ⓓ		(60)

SAMPLE

***** Write Correct Option

Darken Circle if question not attempted

I confirm that all particulars given above are correct.

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Candidate's Signature (in running hand)

Total correct answers (To be filled by the Evaluator)	☐☐
Total correct answers (To be filled by the Coordinator)	☐☐

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SOLUTIONS

1. **(b) :** We have, $\cot\left(\frac{\pi}{2} - 2\cot^{-1}\sqrt{3}\right)$

$$= \cot\left(\frac{\pi}{2} - 2\cot^{-1}\left(\cot\frac{\pi}{6}\right)\right) = \cot\left(\frac{\pi}{2} - 2\cdot\frac{\pi}{6}\right)$$

$$= \cot\left(\frac{\pi}{2} - \frac{\pi}{3}\right) = \cot\frac{\pi}{6} = \sqrt{3}$$

2. **(c) :** $\because$ $f(x)$ is continuous at $x = 2$

$\therefore$ $\lim\limits_{x\to 2} f(x) = f(2) = k$ ...(i)

Now, $\lim\limits_{x\to 2} f(x) = \lim\limits_{x\to 2} \frac{x^3 + x^2 - 16x + 20}{(x-2)^2}$

$$= \lim_{x\to 2} \frac{(x-2)^2(x+5)}{(x-2)^2} = \lim_{x\to 2}(x+5) = 7$$

$\therefore$ From (i), we get $k = 7$

3. **(a) :** We have, $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

On comparing corresponding elements of the matrices, we get

$8 + y = 0$ and $2x + 1 = 5 \Rightarrow y = -8$ and $x = 2$

$\therefore$ $x - y = 2 + 8 = 10$

4. **(c) :** Since, A is a symmetric matrix

$\therefore$ $A^T = A$

$$\Rightarrow \begin{bmatrix} 3 & 2x+3 \\ x-1 & x+2 \end{bmatrix} = \begin{bmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{bmatrix}$$

$\Rightarrow$ $x - 1 = 2x + 3 \Rightarrow x = -4$

5. **(d) :** $f(x) = \cot^{-1}x + x$

$$\Rightarrow f'(x) = \frac{-1}{1+x^2} + 1 \Rightarrow f'(x) = \frac{x^2}{1+x^2} \geq 0, \forall\, x \in R$$

$\therefore$ $f(x)$ is increasing in $(-\infty, \infty)$.

6. **(a) :** Minor of $a_{12}(M_{12}) = \begin{vmatrix} -\sin\theta & \sin\theta \\ -1 & 1 \end{vmatrix}$

$$= -\sin\theta + \sin\theta = 0$$

$\Rightarrow$ Co-factor of $a_{12} = -M_{12} = 0$

Minor of $a_{22}(M_{22}) = \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 1 + 1 = 2$

$\Rightarrow$ Co-factor of $a_{22} = M_{22} = 2$

Minor of $a_{32}(M_{32}) = \begin{vmatrix} 1 & 1 \\ -\sin\theta & \sin\theta \end{vmatrix}$

$$= \sin\theta + \sin\theta = 2\sin\theta$$

$\Rightarrow$ Co-factor of $a_{32} = -M_{32} = -2\sin\theta$

7. **(b) :** Given, $\alpha R\beta \Leftrightarrow \alpha \perp \beta$

$\because$ $\alpha \perp \beta \Leftrightarrow \beta \perp \alpha \therefore \beta R\alpha$

Thus, R is symmetric.

But R is neither reflexive nor transitive, as no line can be $\perp$ to itself. Also, if $\alpha \perp \beta$ and $\beta \perp \gamma$, then $\alpha \parallel \gamma$.

8. **(a) :** Given, $\begin{bmatrix} x\cdot y & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$

$\Rightarrow$ $x \cdot y = 8, w = 4,$

$z + 6 = 0, x + y = 6$

$\Rightarrow$ $z = -6, x + y = 6$

$\therefore$ $x + y + z = 6 - 6 = 0$

9. **(a) :** Here, $\frac{dy}{dx} = 3x^2 + 3$

$\Rightarrow$ Slopes of tangents at $x = 1$ and $x = -1$ are equal.

Hence, the two tangents are parallel.

10. **(c) :** We have, $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) + 4\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$$= \frac{\pi}{3} + 2\cdot\frac{\pi}{6} + 4\cdot\frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} + \frac{2\pi}{3} = \frac{4\pi}{3}$$

11. **(b) :** Let $g(x_1) = g(x_2)$

$\Rightarrow$ $x_1^2 - 4x_1 - 5 = x_2^2 - 4x_2 - 5 \Rightarrow x_1^2 - x_2^2 = 4(x_1 - x_2)$

$\Rightarrow$ $(x_1 - x_2)(x_1 + x_2 - 4) = 0$

Either $x_1 = x_2$ or $x_1 + x_2 = 4$

Either $x_1 = x_2$ or $x_1 = 4 - x_2$

$\therefore$ There are two values of x_1, for which $g(x_1) = g(x_2)$.

$\therefore$ $g(x)$ is not one-one on $x \in R$

12. **(d) :** We have, $y = \log_e\left(\frac{x^2}{e^2}\right)$

$$\therefore \frac{dy}{dx} = \frac{e^2}{x^2}\cdot\frac{1}{e^2}\cdot 2x = \frac{2}{x} \Rightarrow \frac{d^2y}{dx^2} = -\frac{2}{x^2}$$

13. **(c) :** We have, $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 5 & y \\ x & 0 \end{bmatrix}$

Now, $A = A^T \Rightarrow \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix} = \begin{bmatrix} 5 & y \\ x & 0 \end{bmatrix} \Rightarrow x = y$

14. (c) : We have, $x = \frac{2at}{1+t^3}$, $y = \frac{2at^2}{(1+t^3)^2}$

$$\Rightarrow \quad y = \frac{4a^2t^2}{(1+t^3)^2}\cdot\frac{1}{2a} = \frac{x^2}{2a} \Rightarrow \frac{dy}{dx} = \frac{2x}{2a} = \frac{x}{a}$$

15. (a) : We have, $A = \begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$

$\therefore$ Co-factor of $a_{21} = (-1)^{2+1}\begin{vmatrix} 6 & -3 \\ -7 & 3 \end{vmatrix}$

$= -1\,(18 - 21) = 3$

16. (d) : The given parabola is $y^2 = 8x$ ...(i)

$$\Rightarrow \quad 2y\frac{dy}{dx} = 8 \quad \Rightarrow \quad \frac{dy}{dx} = \frac{4}{y}$$

Now, slope of normal $= \left(\frac{-dx}{dy}\right)_{\text{At }(0,\,0)} = -\left(\frac{0}{4}\right) = 0$

Now, equation of normal at (0, 0) is

$y - 0 = 0\,(x - 0)$

$\Rightarrow \quad y = 0$

17. (a) : We have, $|A| = \begin{vmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{vmatrix}$

$= -2(4 - 0) - 0 + 0 = -8$

Now, as we know that if A is non-singular matrix, then $|\text{adj } A| = |A|^{n-1}$, where n is the order of A.

$\therefore \quad |\text{adj } A| = (-8)^2 = 64$

18. (a) : $\log_{10}\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = 2$ (Given)

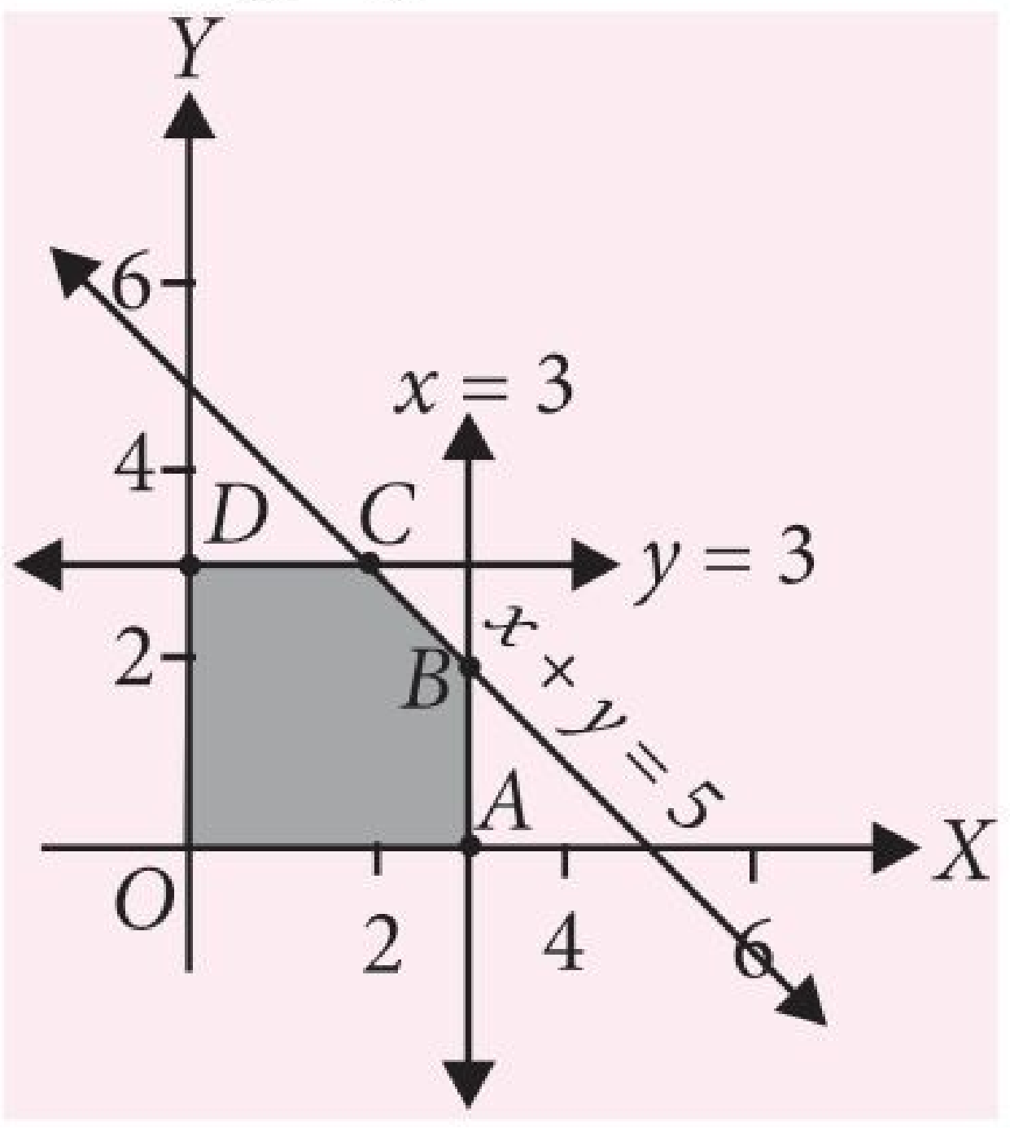
$$\Rightarrow \quad \frac{x^2 - y^2}{x^2 + y^2} = 100 \quad \Rightarrow \quad 99x^2 + 101y^2 = 0$$

Differentiating w.r.t. 'x', we get

$$2(99)x + 2(101)y\frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{99x}{101y}$$

19. (c) : Converting inequations into equations and drawing the graph of corresponding lines,

$x = 3$, $y = 3$ and $x + y = 5$, we get the graph as shown:



As $x \geq 0$, $y \geq 0$, solution lies in first quadrant.

Here, $A(3, 0)$, $B(3, 2)$, $C(2, 3)$, $D(0, 3)$ and $O(0, 0)$ are the corner points.

Now, let us find the value of $Z = 10x + 25y$ at these points.

$\therefore \quad Z(O) = 10(0) + 25(0) = 0$

$Z(A) = 10(3) + 25(0) = 30$

$Z(B) = 10(3) + 25(2) = 80$

$Z(C) = 10(2) + 25(3) = 95$

$Z(D) = 10(0) + 25(3) = 75$

$\therefore$ Z has maximum value 95 at $C(2, 3)$.

20. (c) : We have, $f(x) = \frac{x}{\log x}$, which is defined for $x > 0$ and $x \neq 1$

Also, $f'(x) = \frac{\log x \cdot 1 - x \cdot \frac{1}{x}}{(\log x)^2} = \frac{\log x - 1}{(\log x)^2}$

Clearly, $f'(x) > 0 \Rightarrow \log x > 1 \Rightarrow x > e \;\; \therefore \;\; x \in (e, \infty)$.

21. (d) : Here, $0 \leq \cos^{-1}(\log_2(x^2 + 5x + 8)) \leq \pi$

Also, $-1 \leq \log_2 (x^2 + 5x + 8) \leq 1$

$\Rightarrow \quad 2^{-1} \leq (x^2 + 5x + 8) \leq 2$

$$\Rightarrow \quad \frac{1}{2} \leq (x^2 + 5x + 8) \leq 2$$

$$\Rightarrow \quad x^2 + 5x + \frac{15}{2} \geq 0 \quad \Rightarrow \quad \left(x + \frac{5}{2}\right)^2 + \frac{5}{4} \geq 0$$

and $x^2 + 5x + 6 \leq 0$

$\Rightarrow \; (x + 3)(x + 2) \leq 0 \; \Rightarrow \; x \in [-3, -2]$.

22. (b) : We have, $\log y = m \tan^{-1} x$

Differentiating w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{m}{1 + x^2} \quad \Rightarrow \; y_1(1 + x^2) = my$$

Again differentiating w.r.t. x, we get

$y_2(1 + x^2) + y_1\,(2x) = my_1$

$\Rightarrow \quad (1 + x^2)\,y_2 + (2x - m)\,y_1 = 0$

23. (d) : Maximum value of $Z = px + qy$ at $(15, 15) = 15p + 15q$ and at $(0, 20) = 20\,q$.

According to given condition, we have

$15p + 15q = 20q \;\; \Rightarrow \;\; 15p = 5q \;\; \Rightarrow \;\; q = 3p$

24. (c) : $y = a^{x^{a^{x^{\cdots\infty}}}} \quad \Rightarrow y = a^{x^y}$

Taking log on both sides, we get

$\log y = x^y \log a$

Again, taking log on both sides, we get

$\log(\log y) = \log x^y + \log(\log a) = y \log x + \log(\log a)$

Now, differentiating w.r.t. x, we get

$$\frac{1}{\log\, y}\;\frac{1}{y}\;\frac{dy}{dx} = \frac{y}{x} + \log\, x\;\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{y^2 \log y}{x(1 - y \log x \log y)}\right)$$

MtG

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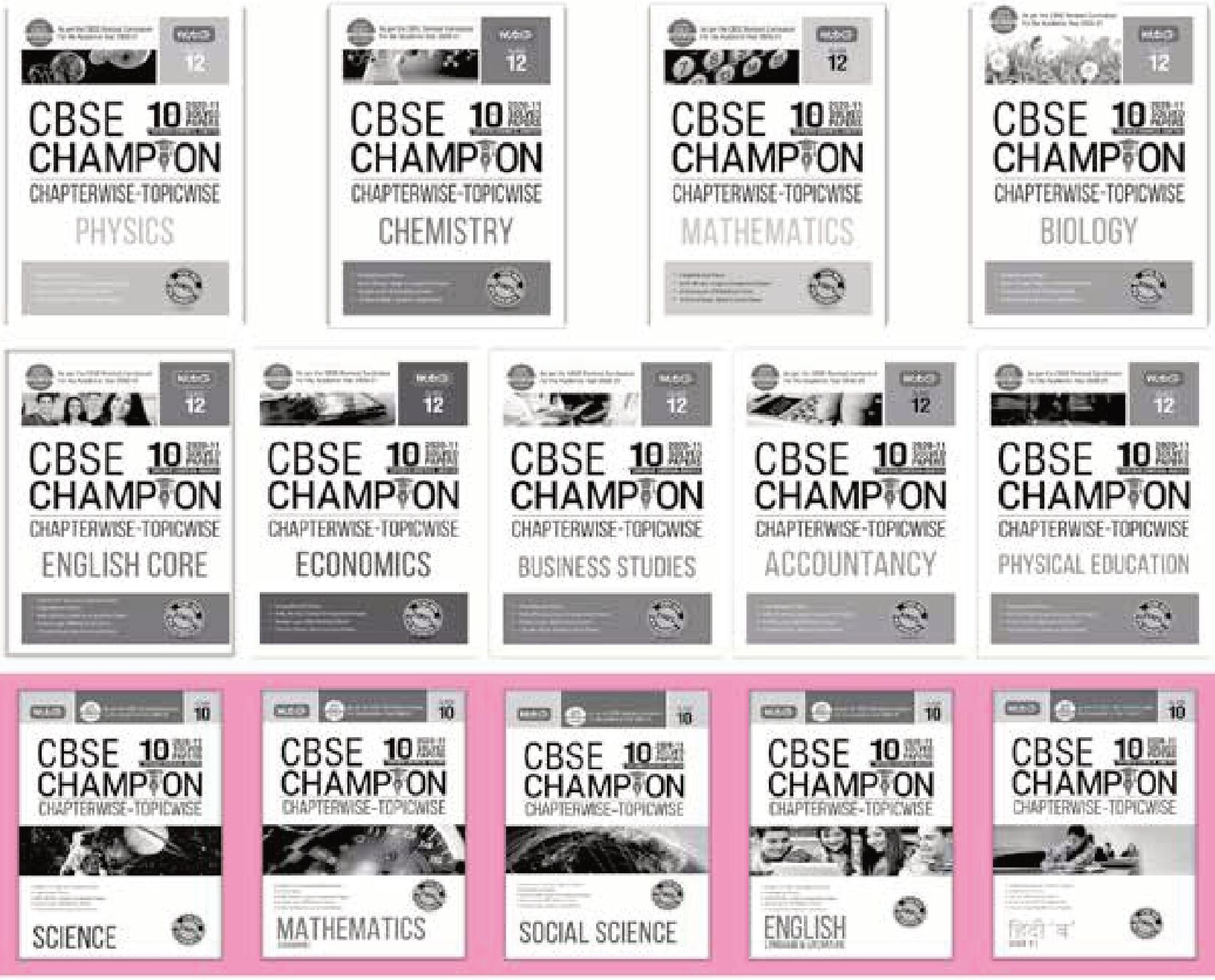
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Questions are supported with topicwise graphical analysis of previous years CBSE Board questions as well as comprehensive and lucid theory. The questions in each topic have been arranged in descending order as per the marks category. Questions from Delhi, All India, Foreign and Compartment papers are included. This ensures that all types of questions that are necessary for Board exam preparation have been covered.

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25. (a) : We have, Area of ΔABC = 3 sq. units

$$\Rightarrow \frac{1}{2}\begin{vmatrix}1 & 3 & 1\\ 0 & 0 & 1\\ k & 0 & 1\end{vmatrix} = \pm 3 \Rightarrow \begin{vmatrix}1 & 3 & 1\\ 0 & 0 & 1\\ k & 0 & 1\end{vmatrix} = \pm 6$$

$\Rightarrow$ $1(0-0)-3(0-k)+1\,(0-0)=\pm 6$

$\Rightarrow$ $3k=\pm 6 \Rightarrow k=\pm 2$

26. (b) : Slope of normal of given curve

$= \text{Slope of the line } (x+14y+4=0) = -\dfrac{1}{14}$

27. (c) : We have, $\sin^{-1}\left(\sin\dfrac{5\pi}{6}\right)+\tan^{-1}\left(\tan\dfrac{\pi}{6}\right)$

$$= \sin^{-1}\left(\sin\left(\pi-\frac{\pi}{6}\right)\right)+\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \sin^{-1}\left(\sin\frac{\pi}{6}\right)+\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \sin^{-1}\left(\frac{1}{2}\right)+\frac{\pi}{6} = \frac{\pi}{6}+\frac{\pi}{6}=\frac{\pi}{3}$$

28. (d) : Minor of 6 $= M_{23} = \begin{vmatrix}1 & 2\\ 7 & 8\end{vmatrix} = 8-14=-6$

Co-factor of 4 $= c_{21} = (-1)^{2+1}\begin{vmatrix}2 & 3\\ 8 & 9\end{vmatrix} = -(18-24)=6$

29. (d) : We have, $f(x)=\dfrac{a\sin x+2\cos x}{\sin x+\cos x}$

$$\Rightarrow f'(x)=\frac{(a\cos x-2\sin x)(\sin x+\cos x)-(a\sin x+2\cos x)(\cos x-\sin x)}{(\sin x+\cos x)^2}$$

$$=\frac{-2\sin^2 x+a\cos^2 x+a\sin^2 x-2\cos^2 x}{(\sin x+\cos x)^2}$$

$$=\frac{a-2}{(\sin x+\cos x)^2}$$

Since, $f(x)$ is strictly increasing for all values of x.

$\Rightarrow$ $f'(x)>0 \Rightarrow a-2>0 \Rightarrow a>2$

$(\because\ (\sin x+\cos x)^2>0)$

30. (d) : We have, $f(-1)=(-1)^2=1$ and $f(1)=1^2=1$.
Thus, two different elements in R have the same image.
$\therefore$ f is not one-one.
If we consider -1 in R (co-domain), then it is clear that there is no element in R (domain) whose image is -1.
$\therefore$ f is not onto. Hence, f is neither one - one nor onto.

31. (b) : We have, $f(x)=\begin{cases}1-x & , \quad x<1\\ (1-x)(2-x) & , \quad 1\le x\le 2\\ 3-x & , \quad x>2\end{cases}$

$\because \lim\limits_{x\to 2^-} f(x)=f(2^-)=(1-2)(2-2)=0$ and

$\lim\limits_{x\to 2^+} f(x)=f(2^+)=(3-2)=1$

$\Rightarrow \lim\limits_{x\to 2^-} f(x)\neq \lim\limits_{x\to 2^+} f(x)$

Hence, the continuity fails at $x=2$.

32. (d) : We have, $A=\begin{bmatrix}2x & 0\\ x & x\end{bmatrix}$, $A^{-1}=\begin{bmatrix}1 & 0\\ -1 & 2\end{bmatrix}$

We know that, $AA^{-1}=I$

$$\Rightarrow \begin{bmatrix}2x & 0\\ x & x\end{bmatrix}\begin{bmatrix}1 & 0\\ -1 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \Rightarrow \begin{bmatrix}2x & 0\\ 0 & 2x\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

On comparing the corresponding elements, we get

$2x=1 \Rightarrow x=\dfrac{1}{2}$

33. (b) : The corner points of the feasible region are $O(0,0)$, $A(4,0)$, $B(2,3)$, $C(0,4)$.
Now, the value of $Z=-3x+4y$ at corner points are
$Z(O)=0$, $Z(A)=-12$, $Z(B)=6$, $Z(C)=16$
$\therefore$ Minimum $Z=-12$ at point A.

34. (b) : Let $f(x)=x^4-62x^2+ax+9$.
Then, $f'(x)=4x^3-124x+a$.
It is given that $f(x)$ attains its maximum at $x=1$
$\therefore$ $f'(1)=0 \Rightarrow 4-124+a=0 \Rightarrow a=120$

35. (b) : $A^T=\begin{bmatrix}0 & -2 & 3\\ 2 & 0 & 1\\ -3 & -1 & 0\end{bmatrix}=-\begin{bmatrix}0 & 2 & -3\\ -2 & 0 & -1\\ 3 & 1 & 0\end{bmatrix}=-A$

Thus, $A^T=-A$, $\Rightarrow A$ is a skew-symmetric matrix.

36. (d) : Let $\cot^{-1}x=\theta \Rightarrow \cot\theta=x \Rightarrow \sin\theta=\dfrac{1}{\sqrt{1+x^2}}$

Now, $\sin(\cot^{-1}x)=\sin\theta=\dfrac{1}{\sqrt{1+x^2}}$

37. (c) : We have, relation aRb is defined iff $|a-b|\le 1$
Clearly, $|a-a|=0<1 \Rightarrow aRa \Rightarrow R$ is reflexive.
Also, $|a-b|\le 1 \Rightarrow |b-a|\le 1$, *i.e.*, $aRb \Rightarrow bRa$
$\Rightarrow$ R is symmetric.
Thus, R is both reflexive and symmetric.

38. (a) : We have, $A=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$

$$\therefore \text{adj } A=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}^T=\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

39. (c) : We have, $y=x^2-\dfrac{1}{x^2}$

$$\therefore \frac{dy}{dx}=2x-\frac{(-2)}{x^3}=2x+\frac{2}{x^3}$$

Now, $\left[\frac{dy}{dx}\right]_{(-1,0)} = -2 + \frac{2}{(-1)} = -2 - 2 = -4$

$\therefore$ Slope of normal $= \frac{-1}{\left[\frac{dy}{dx}\right]_{(-1,0)}} = \frac{-1}{-4} = \frac{1}{4}$

40. (b) : Given, $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$, $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$

Now, $A^2 = A \cdot A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & 2ab \\ 2ab & a^2 + b^2 \end{bmatrix}$

$\Rightarrow \alpha = a^2 + b^2, \beta = 2ab$

41. (a) :

Corner Points	Value of $Z = x + y$
$O(0, 0)$	0
$A(25, 0)$	25
$B\left(\frac{50}{3}, \frac{40}{3}\right)$	30 (Maximum)
$C(0, 20)$	20

$\therefore$ Maximum value of Z occurs at a unique point.

42. (c) : The given curve is $y^2 e^{xy} = 9e^{-3}x^2$

Taking log on both sides, we get

$2\log y + xy = \log 9 - 3 + 2\log x$ ...(i)

Differentiating (i) w.r.t. x, we get

$\frac{2}{y}\frac{dy}{dx} + x\frac{dy}{dx} + y = \frac{2}{x} \Rightarrow \frac{dy}{dx}\left(\frac{2}{y} + x\right) + y = \frac{2}{x}$

Now, $\left[\frac{dy}{dx}\right]_{(-1,3)}\left(\frac{2}{3} + (-1)\right) + 3 = \frac{2}{-1}$

$\Rightarrow \left[\frac{dy}{dx}\right]_{(-1,3)}\left(\frac{-1}{3}\right) = -5 \Rightarrow \left[\frac{dy}{dx}\right]_{(-1,3)} = 15$

43. (c) : Let $f(x) = y = x^3 + x^2 + x + 1$

Then, $f'(x) = 3x^2 + 2x + 1$.

For a maximum or minimum, put

$f'(x) = 0 \Rightarrow 3x^2 + 2x + 1 = 0$

But, this equation gives imaginary values of x.

So, $f'(x) \neq 0$ for any real value of x.

Hence, $f(x)$ neither have a local maximum nor local minimum.

44. (d) : Maximum of objective function occurs at corner points.

Corner Points	Value of $Z = 2x + 5y$
(0, 0)	0
(7, 0)	14
(6, 3)	27
(4, 5)	33 (Maximum)
(0, 6)	30

Thus, maximum of Z, occurs at (4, 5).

45. (d) : For singular matrix, $|A| = 0$

$\therefore \begin{vmatrix} 1 & 2 & x \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 0$

$\Rightarrow 1(-1 - 1) - 2(-1 - 2) + x(1 - 2) = 0$

$\Rightarrow -2 + 6 - x = 0 \Rightarrow x = 4$

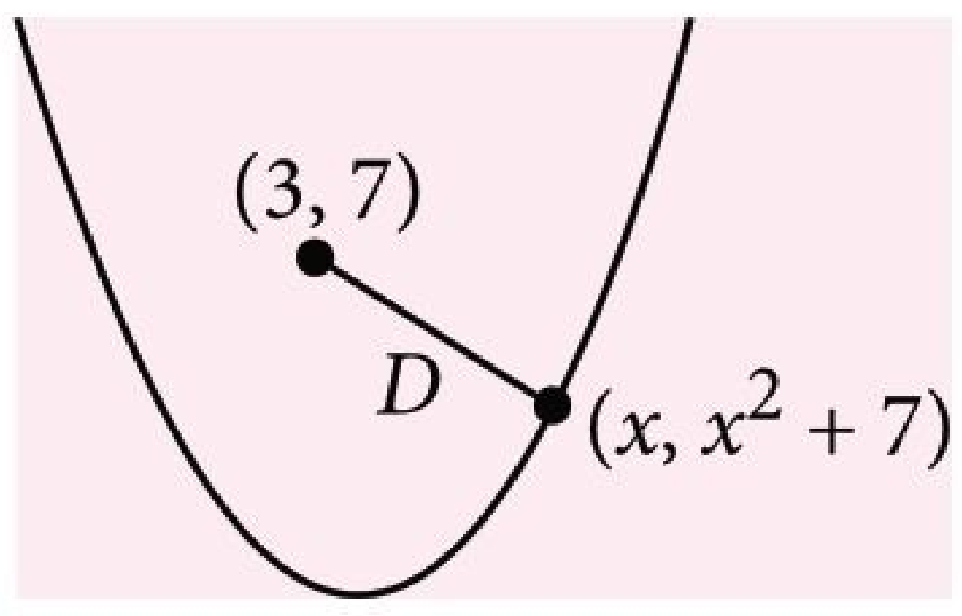
46. (c) : For all values of x, $y = x^2 + 7$

$\therefore$ Arun's position at any point will be given by $(x, x^2 + 7)$

47. (c) : Distance between Arun and Manita, *i.e.*, D

$= \sqrt{(x-3)^2 + (x^2 + 7 - 7)^2}$

$= \sqrt{(x-3)^2 + x^4}$



48. (a) : We have, $D = \sqrt{(x-3)^2 + x^4}$

$\therefore D^2 = (x - 3)^2 + x^4$

Now, $\frac{d}{dx}(D^2) = 2(x - 3) + 4x^3 = 0$

$\Rightarrow 4x^3 + 2x - 6 = 0 \Rightarrow 2x^3 + x - 3 = 0$

$\Rightarrow (x - 1)(2x^2 + 2x + 3) = 0 \quad \therefore x = 1$

($\because 2x^2 + 2x + 3 = 0$ will give imaginary values)

49. (b) : We have, $D^2 = (x-3)^2 + x^4$

and $\frac{d}{dx}(D^2) = 4x^3 + 2x - 6 = 0 \Rightarrow x = 1$

Also, $\left[\frac{d^2}{dx^2}(D^2)\right]_{x=1} = [12x^2 + 2]_{x=1} = 14 > 0$

$\therefore$ D^2 is minimum when $x = 1$.

$\therefore$ Value of x for which D will be minimum is 1

For $x = 1$, $y = 8$.

Thus, the required position is (1, 8).

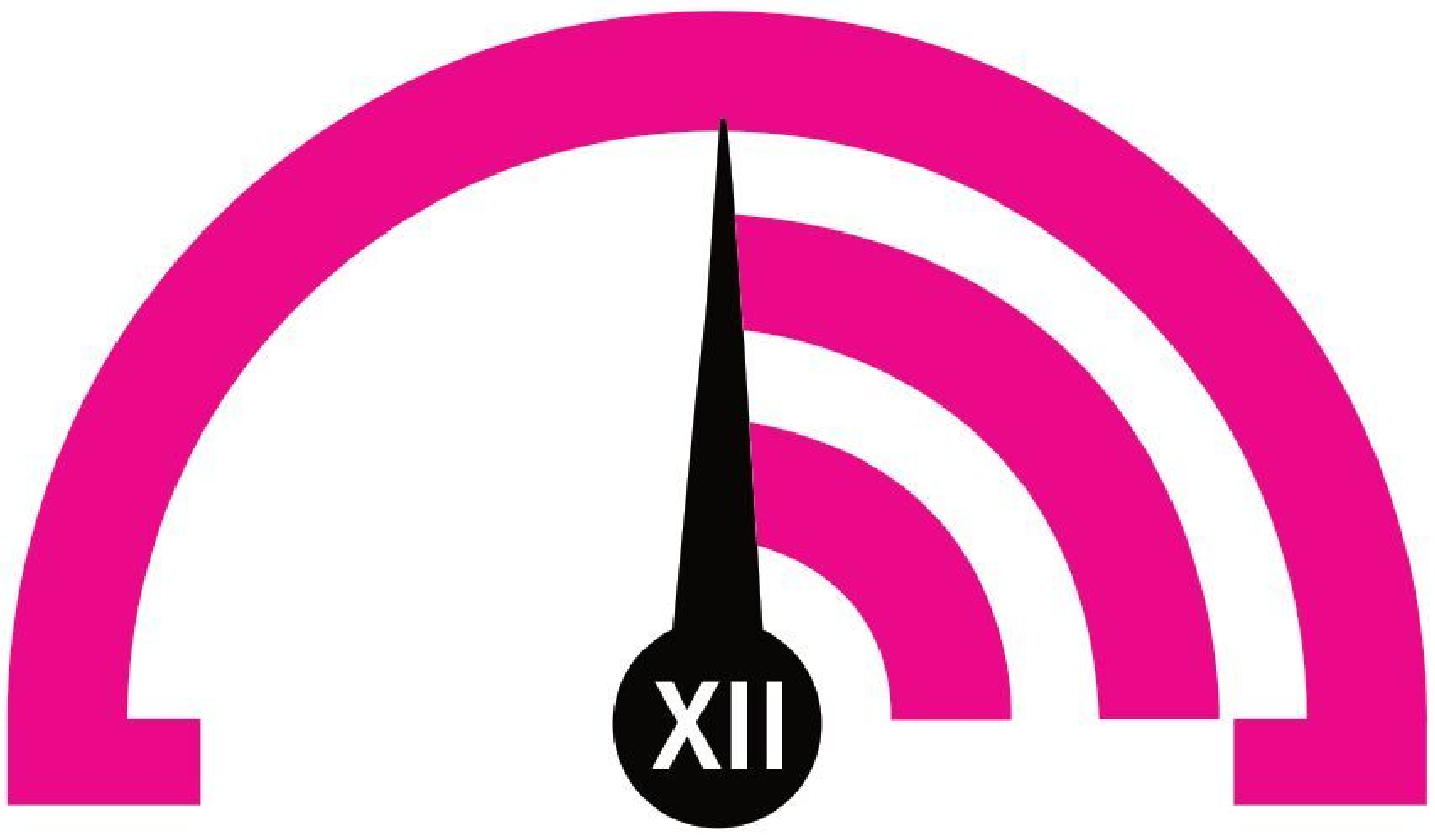
Arun
Manita
(1, 8)
(3, 7)

50. (d) : Minimum value of $D = \sqrt{(1-3)^2 + (8-7)^4}$

$= \sqrt{4 + 1} = \sqrt{5}$

◆ ◆

MONTHLY TEST DRIVE



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Total Marks : 80

Series 6 : Indefinite Integrals

Time Taken : 60 Min.

Only One Option Correct Type

1. $\int x^2 \sin 2x\, dx =$

(a) $-\frac{x^2}{2}\sin 2x + \frac{x}{2}\sin 2x + \frac{\cos 2x}{4} + c$

(b) $-\frac{x^2}{2}\sin 2x + \frac{x}{2}\cos 2x - \frac{\sin 2x}{4} + c$

(c) $\frac{x^2}{2}\cos 2x - \frac{x}{2}\sin 2x + \cos 2x + c$

(d) $-\frac{x^2}{2}\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + c$

2. If $\int \frac{(3x+2)^{10}}{(4x+3)^{12}}\, dx = A\left(\frac{3x+2}{4x+3}\right)^B + C$, then $AB =$

(a) 1 (b) 10 (c) 11 (d) 12

3. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} \cdot \frac{dx}{x} =$

(a) $\ln\left(\frac{1-\sqrt{1-x}}{\sqrt{x}}\right) - \sin^{-1}\sqrt{x} + c$

(b) $2\ln\left(\frac{1-\sqrt{1-x}}{\sqrt{x}}\right) - \cos^{-1}\sqrt{x} + c$

(c) $2\ln\left(\frac{1-\sqrt{1-x}}{\sqrt{x}}\right) - 2\sin^{-1}\sqrt{x} + c$

(d) $2\ln\left(\frac{1-\sqrt{1-x}}{\sqrt{x}}\right) - 2\cos^{-1}\sqrt{x} + c$

4. $\int \tan x \tan 2x\, dx =$

(a) $\frac{1}{2}\ln\tan\left|\frac{\pi}{4} + x\right| + x + c$

(b) $\frac{1}{2}\ln\tan\left|\frac{\pi}{4} + x\right| - x + c$

(c) $\frac{1}{2}\ln\tan\left|\frac{\pi}{4} - x\right| + x + c$

(d) $\frac{1}{2}\ln\tan\left|\frac{\pi}{4} - x\right| - x + c$

5. $\int \left(x^{3m} + x^{2m} + x^m\right)\left(2x^{2m} + 3x^m + 6\right)^{\frac{1}{m}} dx =$

(a) $\frac{1}{6m}(2x^{3m} + 3x^{2m} + 6x^m)^{1/m} + c$

(b) $\frac{1}{6(m+1)}(2x^{3m} + 3x^{2m} + 6x^m)^{(m+1)/m} + c$

(c) $\frac{1}{6m}(2x^{3m} + 3x^{2m} + 6x^m)^{m+1/m} + c$

(d) $\frac{1}{6(m+1)}(2x^{3m} + 3x^{2m} + 6x^m)^{1/m} + c$

6. Let $f(x)$ be an even function, $I_1 = \int_0^{\frac{\pi}{2}} f(\cos 2x)\cos x\, dx$, $I_2 = \int_0^{\frac{\pi}{4}} f(\sin 2x)\cos x\, dx$. Then $\frac{I_1}{I_2} =$

(a) 1 (b) 1/2 (c) $1/\sqrt{2}$ (d) $\sqrt{2}$

One or More than One Option(s) Correct Type

7. If $\int \sqrt{\frac{\cos x - \cos^3 x}{1-\cos^3 x}}\, dx = f(x) + c$, then $f(x) =$

(a) $3/2 \sin^{-1}(\cos^{3/2}x)$ + Real constant

(b) $2/3 \cos^{-1}(\cos^{3/2}x)$ + Real constant

(c) $-2/3 \sin^{-1}(\cos^{3/2}x)$ + Real constant

(d) $2/3 \sin^{-1}(\cos^{2/3}x) + c$ + Real constant

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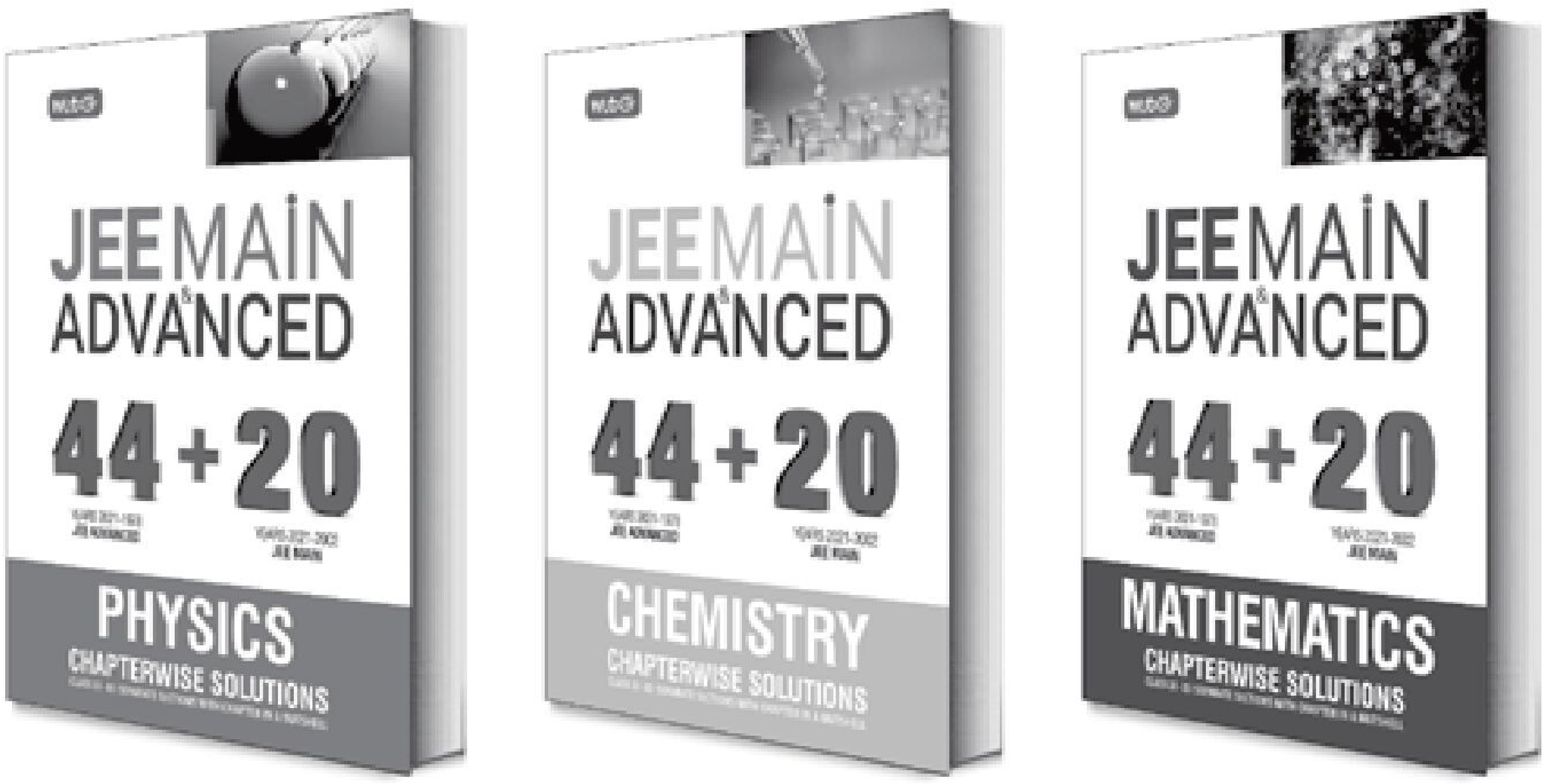
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8. Let $A(x)=\int\frac{dx}{e^{3x}+15e^{-3x}+9e^{-9x}}$,

$B(x)=\int\frac{dx}{e^{9x}+15e^{3x}+9e^{-3x}}$ and

$M(x) = A(x) - 3B(x)$. If $M(x)=\frac{1}{\lambda}\tan^{-1}(f(x))+C$, $C\in R$ then

(a) $\lambda = 9$ (b) $f(x)=\frac{e^{3x}+3e^{-3x}}{3}$

(c) $24 f(\log 2) = 67$ (d) $27 f(\log 3) = 244$

9. $\int\frac{(\sqrt{1-x^2}+x)}{\sqrt{1-x^2}\,(1+x\sqrt{1-x^2})}dx$ equals

(a) $\frac{1}{\sqrt{3}}\log\left|\frac{\sqrt{3}+x-\sqrt{1-x^2}}{\sqrt{3}+\sqrt{1-x^2}-x}\right|+C$

(b) $\frac{1}{3}\log\left|\frac{\sqrt{3}+x-\sqrt{1-x^2}}{\sqrt{3}+\sqrt{1-x^2}-x}\right|+C$

(c) $-\frac{1}{3}\log\left|\frac{\sqrt{3}+x-\sqrt{1-x^2}}{\sqrt{3}+\sqrt{1-x^2}-x}\right|+C$

(d) $-\frac{1}{\sqrt{3}}\log\left|\frac{\sqrt{3}+\sqrt{1-x^2}-x}{\sqrt{3}+x-\sqrt{1-x^2}}\right|+C$

10. If $\int\frac{\log x}{x^4}dx=\frac{-\log x}{ax^3}+\frac{-1}{bx^3}+C$ then

(a) $a = b^2$ (b) $b = a^2$

(c) $b = 3a$ (d) $a^2 + b^2 = 90$.

11. If $\int\frac{\cos 13x-\cos 14x}{1+2\cos 9x}dx=\frac{\sin 4x}{a}-\frac{\sin 5x}{b}+C$, then

(a) $b^2 - a^2 = 3^2$

(b) If $a^b = k^{10} = 32^k$, then $k = 2$

(c) $b^a = 625$

(d) $a^b - b^a + 1 = a^2b^2$

12. If $\int\frac{4x^3+3x^2}{x^8+2x^7+x^6+4x^4+4x^3+13}dx$

$=\frac{k}{k_1}\tan^{-1}(Ax^4+Bx^3+C)+D$, then

(a) $k_1 + k = 4$ (b) $A + B = C$

(c) $k_1 + k = 3$ (d) $A + B = 2C$

Comprehension Type

Paragraph for Q. No. 13 to 15

In some of the cases we can split the integrand into the sum of the two functions such that the integration of one of them by parts produces an integral which cancels the other integral.

13. $\int\left(\frac{1}{\ln x}-\frac{1}{(\ln x)^2}\right)dx$ =

(a) $\ln(\ln x) + c$ (b) $x\ln x + c$

(c) $\frac{x}{\ln x}+c$ (d) $x + \ln x + c$

14. $\int\frac{xe^x}{(1+x)^2}dx$ =

(a) $xe^x + c$ (b) $\frac{e^x}{(x+1)^2}+c$

(c) $e^x-\frac{1}{x+1}+c$ (d) $\frac{e^x}{x+1}+c$

15. The integral of $\int e^{\sin x}(x\cos x-\sec x\tan x)\,dx$ is

(a) $xe^{\sin x}-e^{\sin x}\cdot\sec x+c$

(b) $(x+\sec x)e^{\sin x}+c$

(c) $e^{\sin x}\cos x+c$

(d) $e^{\sin x}(\cos x-\sec x)+c$

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Matrix Match Type

16. Match the following :

	Column-I		Column-II
P.	$\int \frac{\sin x}{\sin x - \cos x} dx =$	1.	$\frac{-\sin 2x}{2} + c$
Q.	$\int (\ln x)^2 dx =$	2.	$\frac{x}{2} + \frac{1}{2} \ln\lvert\sin x - \cos x\rvert + c$
R.	$\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx =$	3.	$x[(\ln x)^2 - 2\ln x + 2] + c$

	P	Q	R		P	Q	R
(a)	1	2	3	(b)	2	3	1
(c)	3	2	1	(d)	2	1	3

Numerical Answer Type

17. If $\int \frac{(3x^2 + 2x)\, dx}{x^6 + 2x^5 + x^4 + 2x^3 + 2x^2 + 5}$

$= \frac{1}{A} \tan^{-1}\left(\frac{x^B + x^2 + C}{D}\right) + K$, then the value of $A + B + C + D$ is ________.

18. If $\int (x^9 + x^6 + x^3)(2x^6 + 3x^3 + 6)^{1/3} dx$

$= \frac{1}{A}(2x^9 + 3x^6 + 6x^3)^B + K$, then the value of $\frac{AB}{8}$ is ________.

19. If $\int |x| \log |x| dx \;\; (x \neq 0)$ is equal to $k_1 x \mid x \mid \log x + k_2 x \mid x \mid + c$, then the value of $\left|\frac{1}{k_1} + \frac{1}{k_2}\right|$ equals ________.

20. If $\int \sin^4 x \, e^{\log \cos x} dx = \frac{1}{k} \sin^p x + c$, then $k =$ ________.

◆◆

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No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

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90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
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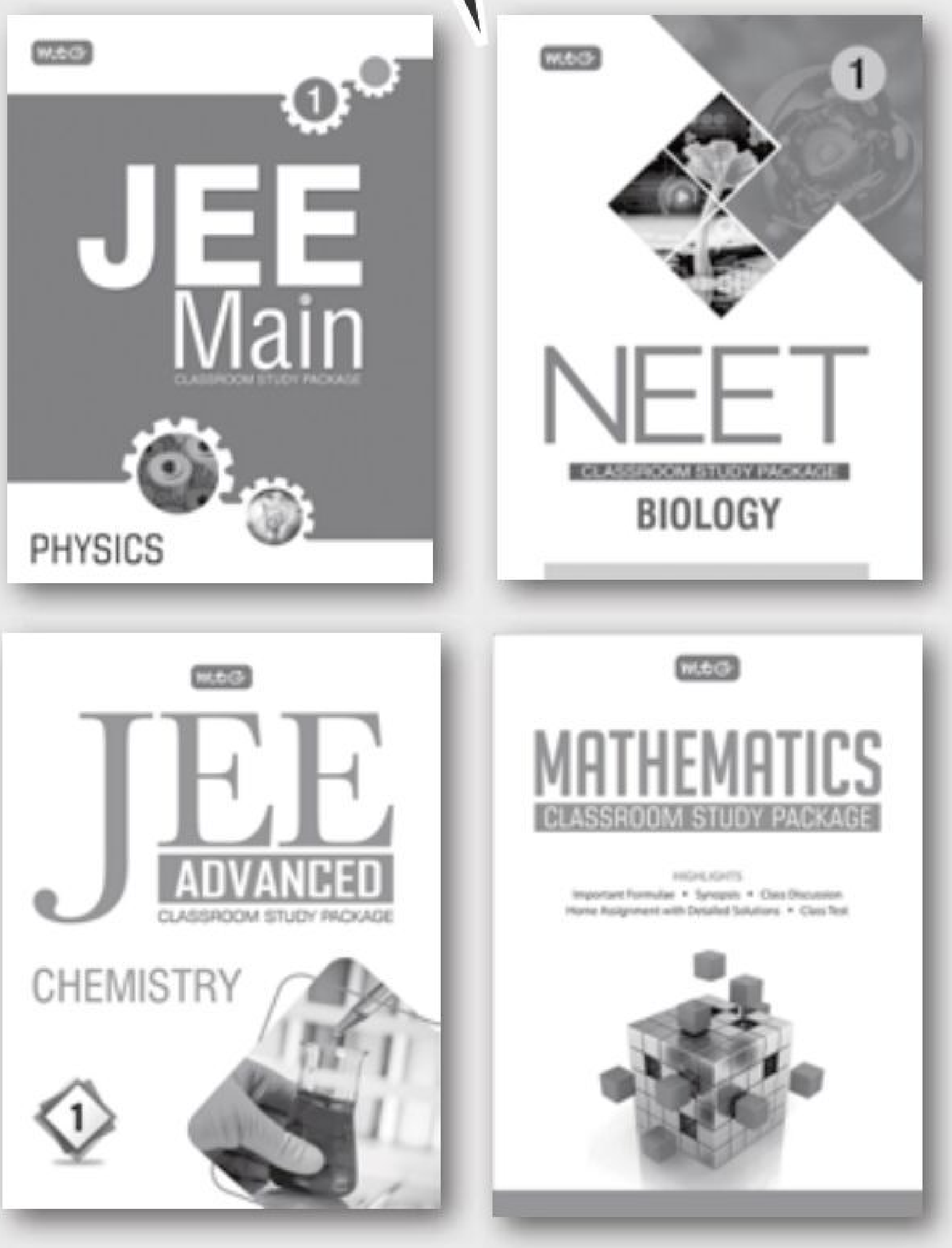

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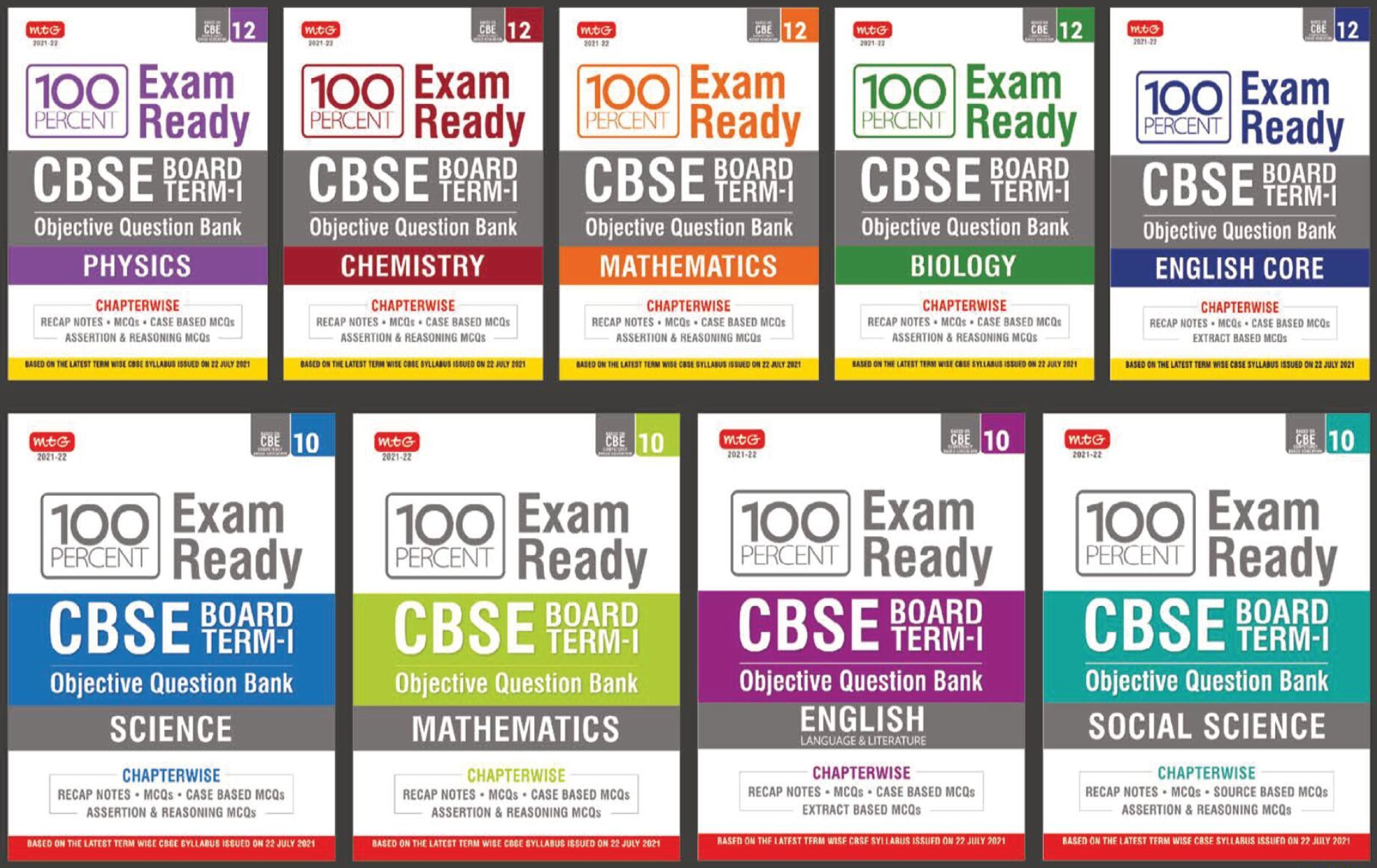
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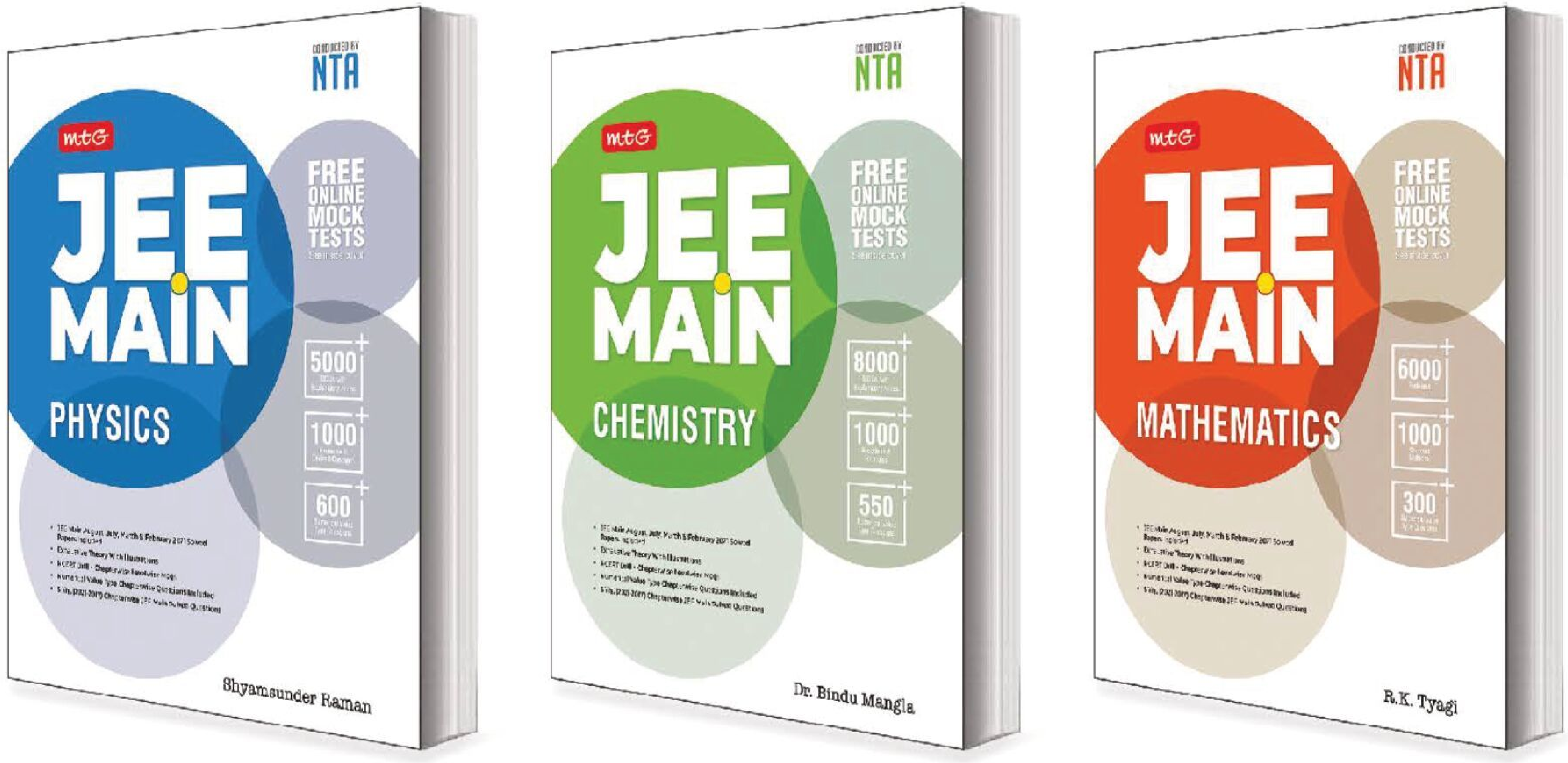
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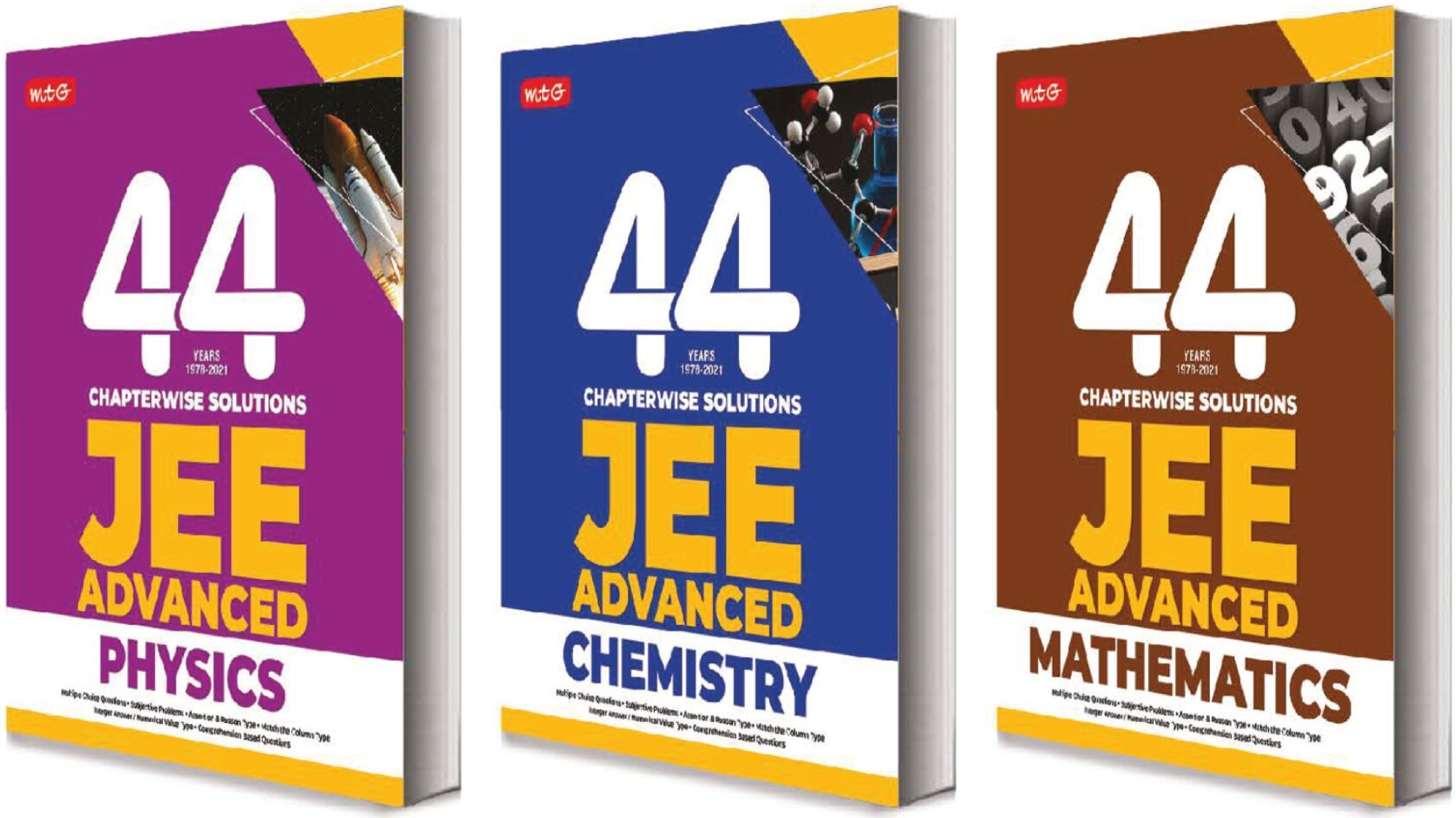

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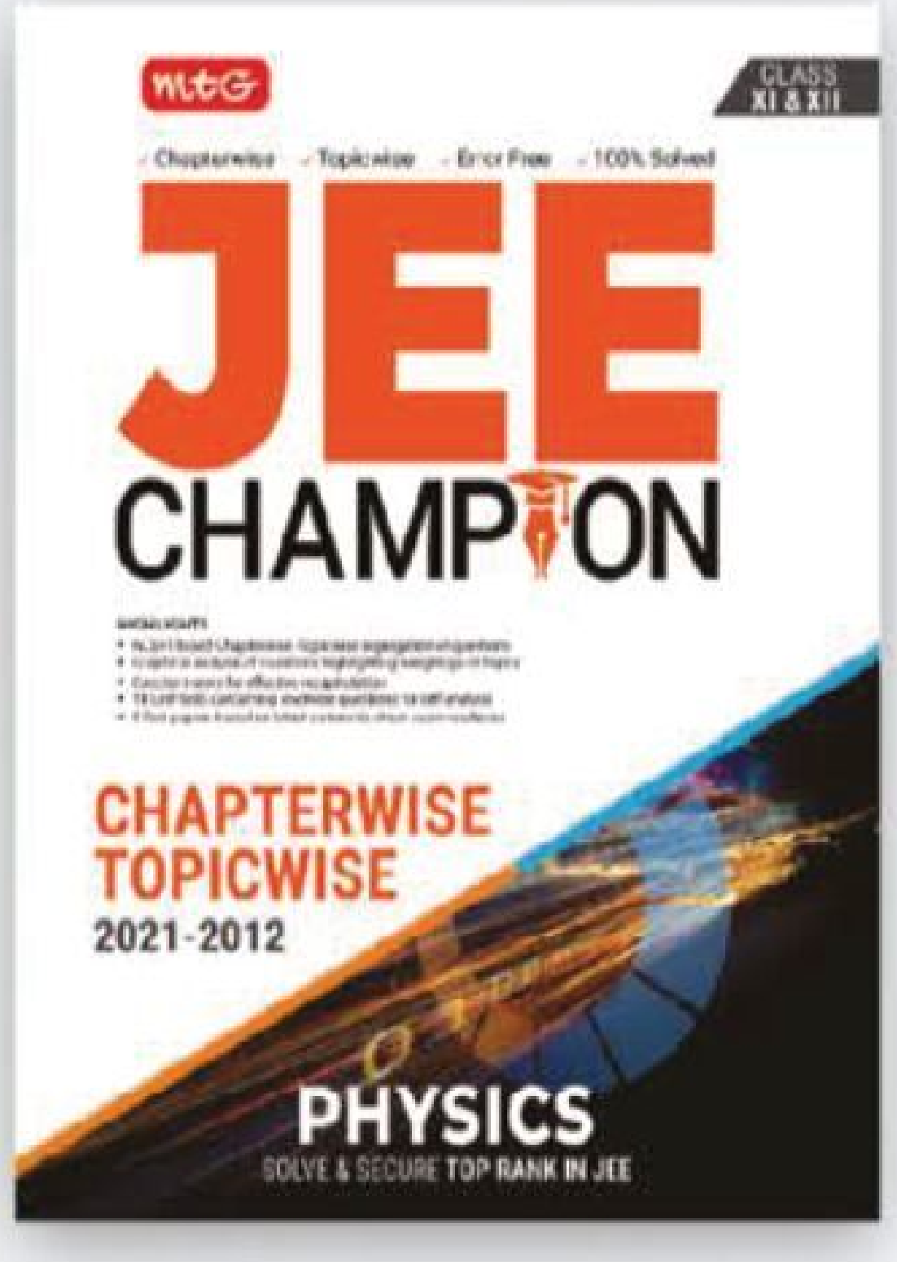
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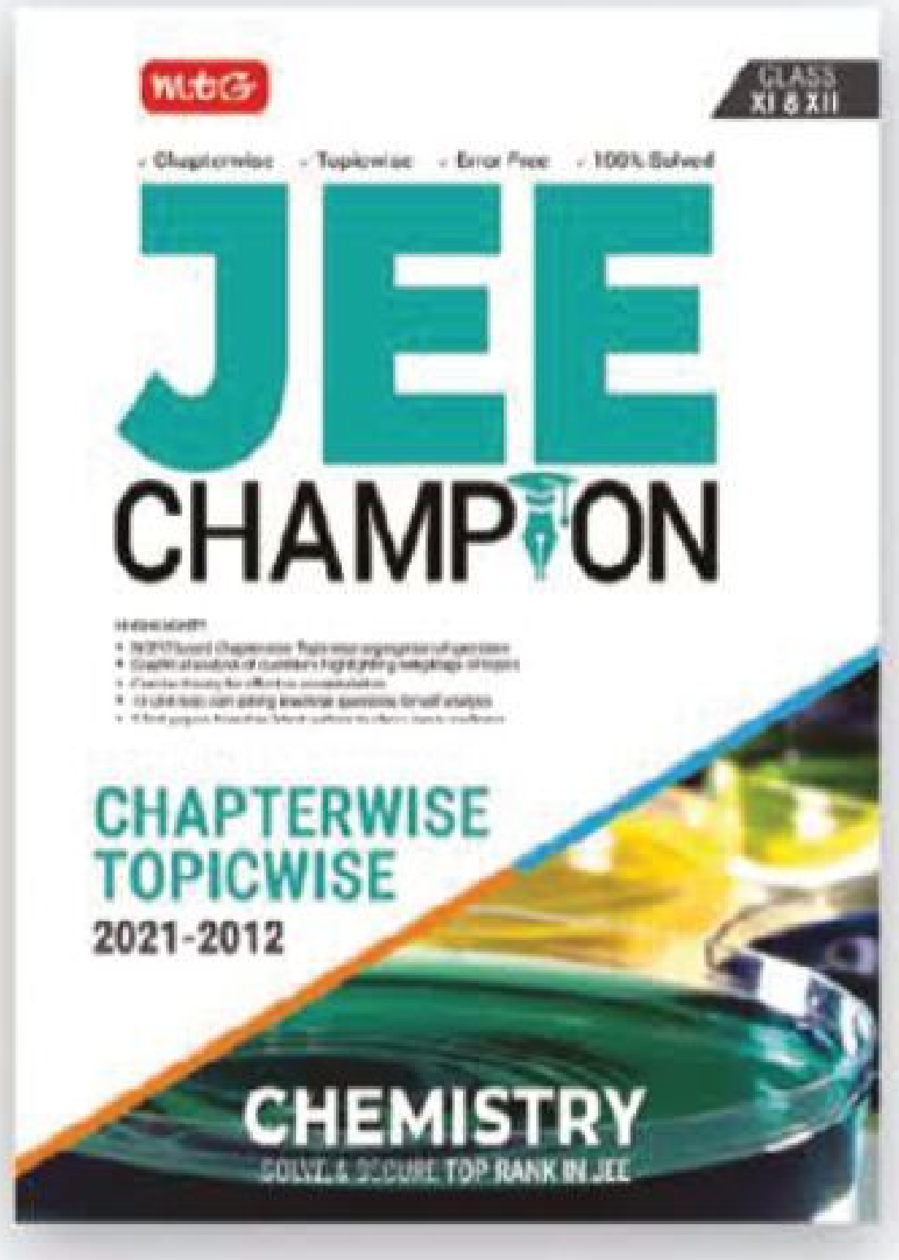
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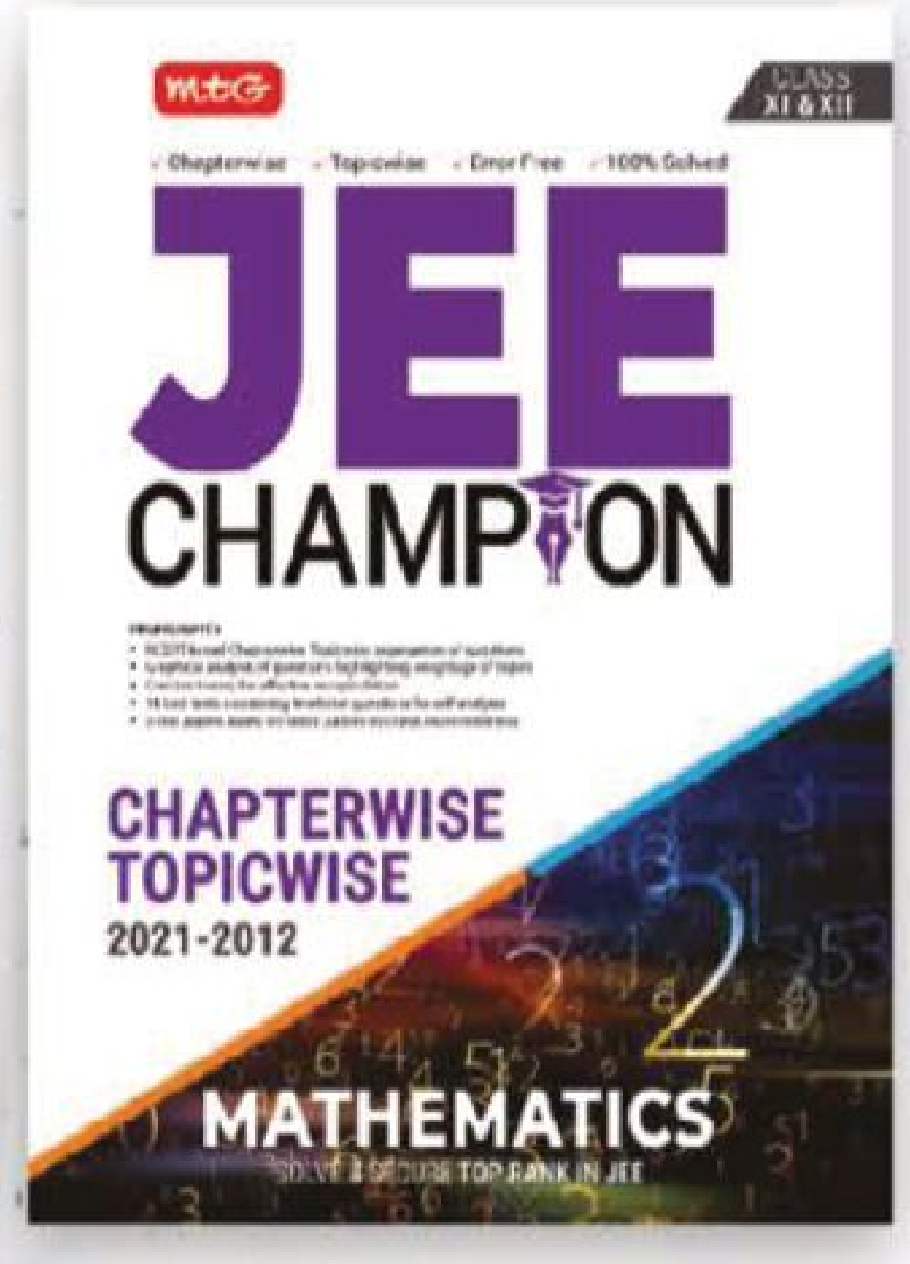
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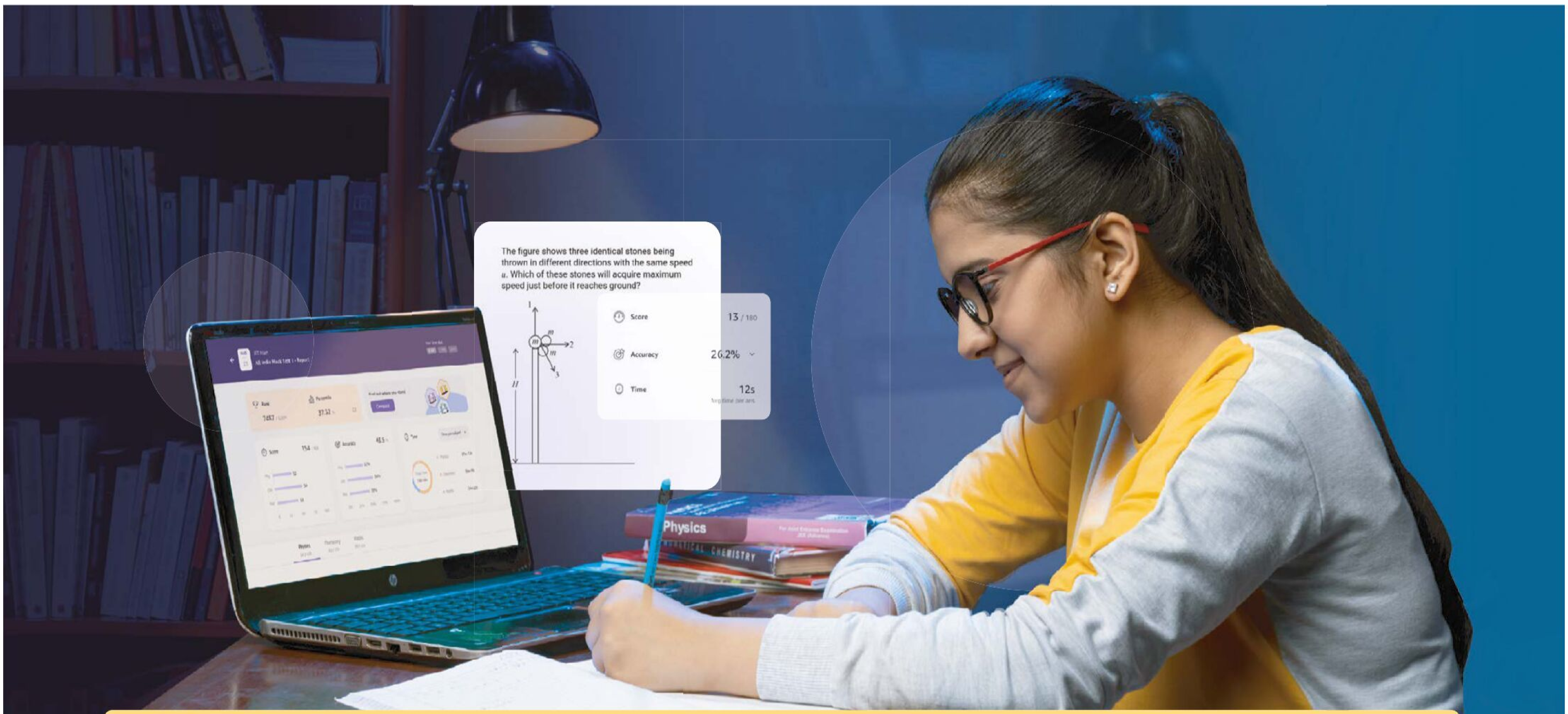
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